Sheet (6)
Operations on the real numbers

We know that 2x and 3x are two like algebraic terms, then 2x + 3x = 5x

then we deduce that $2\sqrt{5} + 3\sqrt{5} = (2+3)\sqrt{5} = 5\sqrt{5}$,

but 2x and 3y are two unlike algebraic terms then their sum 2x + 3y

then the sum of $2\sqrt{3}, 3\sqrt{2}$ written in the form $2\sqrt{3} + 3\sqrt{2}$

Properties of addition of real numbers

Closure :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

i.e. The sum of any two real numbers is a real number, therefore we say $\mathbb{R}$ is closed under addition operation.

For example :

- $\sqrt{5} \in \mathbb{R}$ and $2\sqrt{5} \in \mathbb{R}$ we find that : $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5} \in \mathbb{R}$

Commutative property :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

For exomple :

$5\sqrt[3]{2} + 4\sqrt[3]{2} = 9\sqrt[3]{2}$, $4\sqrt[3]{2} + 5\sqrt[3]{2} = 9\sqrt[3]{2}$

i.e. $5\sqrt[3]{2} + 4\sqrt[3]{2} = 4\sqrt[3]{2} + 5\sqrt[3]{2}$

Associative property :

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

For example :

$(\sqrt{3} + 2\sqrt{3}) + 5\sqrt{3} = 3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$,

$\sqrt{3} + (2\sqrt{3} + 5\sqrt{3}) = \sqrt{3} + 7\sqrt{3} = 8\sqrt{3}$

i.e. $(\sqrt{3} + 2\sqrt{3}) + 5\sqrt{3} = \sqrt{3} + (2\sqrt{3} + 5\sqrt{3})$

The additive neutral :

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example : $\sqrt{2} + 0 = 0 + \sqrt{2} = \sqrt{2}$, $-\sqrt[3]{5} + 0 = 0 + (-\sqrt[3]{5}) = -\sqrt[3]{5}$

The additive inverse of every real number :

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

For example :

- The additive inverse of the number $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + (-\sqrt{3}) = 0$
- The additive inverse of the number $2 + \sqrt{5}$ is $-(2 + \sqrt{5})$ and equals $-2 - \sqrt{5}$
- The additive inverse of the number $3 - \sqrt{2}$ is $-(3 - \sqrt{2})$ and equals $\sqrt{2} - 3$
- The additive inverse of the number zero is itself.

[1] Find the result of each of the following in the simplest form:

(1)	$\sqrt{3} + 2\sqrt{3}$ = ...
(2)	$3\sqrt{2} - 5\sqrt{2}$ = ...
(3)	$2\sqrt{5} - 3\sqrt{5} + \sqrt{5}$ = ...
(4)	$5\sqrt[3]{7} - 8\sqrt[3]{7} + 2\sqrt[3]{7}$ = ...
(5)	$4\sqrt{5} - 2\sqrt{5} + 5\sqrt{5} - \sqrt{5}$ = ...
(6)	$5\sqrt{3} - 7\sqrt{3} + 3\sqrt{3} - \sqrt{3}$ = ...
(7)	$\sqrt{5} - \sqrt{3} + 2\sqrt{5} + \sqrt{3}$ = ...
(8)	$2\sqrt{3} + 5 + \sqrt{3} - 6$ = ...

(9)	$2\sqrt{7}-3\sqrt{2}+\sqrt{7}+5\sqrt{7}$ = ...
(10)	$2\sqrt{2}-3\sqrt[3]{2}+5\sqrt{2}+\sqrt[3]{2}$ = ...
(11)	$8\sqrt{\frac{1}{4}}+2\sqrt[3]{3}-\sqrt[3]{64}-5\sqrt[3]{3}$ = ...
(12)	$\frac{1}{4}\sqrt{2}+\frac{2}{7}\sqrt{5}+\frac{3}{4}\sqrt{2}-\frac{2}{7}\sqrt{5}$ = ...

The properties of multiplication operation of real numbers

Closure :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$

i.e. The product of any two real numbers is a real number therefore we say :

The multiplication operation is closed in $\mathbb{R}$

For example :

- $\sqrt{3} \in \mathbb{R}$ and $2\sqrt{3} \in \mathbb{R}$

  We find that : $\sqrt{3} \times 2\sqrt{3} = 2 \times 3 = 6 \in \mathbb{R}$

Commutative property :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$

For example :

- $2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$, $3\sqrt{5} \times 2\sqrt{5} = 6 \times 5 = 30$

  i.e. $2\sqrt{5} \times 3\sqrt{5} = 3\sqrt{5} \times 2\sqrt{5}$

The associative property :

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$

For example :

- $(2\sqrt{7} \times 4\sqrt{7}) \times \sqrt{7} = 56 \times \sqrt{7} = 56\sqrt{7}$
- $2\sqrt{7} \times (4\sqrt{7} \times \sqrt{7}) = 2\sqrt{7} \times 28 = 56\sqrt{7}$

  i.e. $(2\sqrt{7} \times 4\sqrt{7}) \times \sqrt{7} = 2\sqrt{7} \times (4\sqrt{7} \times \sqrt{7})$

The multiplicative neutral :

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$

i.e. One is the multiplicative neutral in $\mathbb{R}$

For example :

- $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number :

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

For example :

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$ because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $-\frac{\sqrt{2}}{5}$ is $-\frac{5}{\sqrt{2}}$

Notice that :

Both the number and its multiplicative inverse have the same sign.

- The multiplicative inverse of the number 1 is itself and also the multiplicative inverse of -1 is itself.

Notice that :

There is no multiplicative inverse for the number zero because $\frac{1}{\text{zero}}$ is meaningless (**i.e.** undefined)

Remark

- Since each non-zero real number has a multiplicative inverse then the division operation by any real number does not equal zero is possible in $\mathbb{R}$ and it is defined as

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

  i.e. The division operation $(a \div b)$ means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

Then we can deduce that :

The division operation in $\mathbb{R}$ is not commutative and it is not associative.

[2] Find the result of each of the following in the simplest form:

(1)	$\sqrt{3} \times \sqrt{3}$ = ...
(2)	$-2\sqrt{5} \times 3\sqrt{5}$ = ...
(3)	$2 \times 3\sqrt{2}$ = ...
(4)	$\frac{1}{3}\sqrt{3} \times \sqrt{3}$ = ...
(5)	$(\sqrt[3]{5})^3 \times 3\sqrt{3}$ = ...
(6)	$2\sqrt{3} \times \frac{2\sqrt{7}}{7} \div \frac{20\sqrt{3}}{5\sqrt{7}}$ = ...

[3] Make the denominator in each of the following an integer:

(1)	$\frac{3}{\sqrt{3}}$ = ...
(2)	$\frac{10}{\sqrt{5}}$ = ...
(3)	$\frac{-6}{\sqrt{3}}$ = ...
(4)	$\frac{6}{2\sqrt{3}}$ = ...
(5)	$\frac{\sqrt{2}+3}{\sqrt{2}}$ = ...

Distributing multiplication on addition and subtraction

For any three real numbers a , b and c it will be :

- $a(b \pm c) = ab \pm ac$
- $(b \pm c)a = ba \pm ca$

Remarks:

- $(a+b)(a-b) = a^2 - b^2$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$

[4] Find the result of each of the following in the simplest form:

(1)	$2(\sqrt{2}+\sqrt{5}) =$ ....
(2)	$\sqrt{2}(5+\sqrt{2}) =$ ....
(3)	$\sqrt{7}(\sqrt{7}+2) =$ ....
(4)	$-\sqrt{3}(-5-\sqrt{3})$
(5)	$-2\sqrt{5}(3-\sqrt{5}) =$ ....
(6)	$\sqrt{7}\left(\frac{2}{\sqrt{7}}-\sqrt{7}+3\right) =$ ....

[5] Find the result of each of the following operations:

(1)	$(\sqrt{2}+1)(\sqrt{2}-1) =$ ....
(2)	$(4-3\sqrt{2})(4+3\sqrt{2}) =$ ....
(3)	$(\sqrt{5}-1)^2 =$ ....
(4)	$(2\sqrt{3}+4)^2 =$ ....

[6] Complete the following:

(1)	The multiplicative neutral in $\mathbb{R}$ is .... and the additive neutral in $\mathbb{R}$ is ....
(2)	The additive inverse of the number $1-\sqrt{2}$ is ....
(3)	The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\text{.........}}{6}$
(4)	The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\text{.........}}{\sqrt{3}}$
(5)	If : $a=\sqrt{5}$ and $b=2\sqrt{5}$, then : $ab=$ ....
(6)	If : $x=\sqrt{5}+2$ and $y=\sqrt{5}-2$ then $(x+y)^2=$ ....
(7)	If : $x=2\sqrt[3]{5}$, then $x^3=$ ....
(8)	The solution set of the equation : $x^2+25=0$ in $\mathbb{R}$ is ....
(9)	$\mathbb{R}_+ \cup [-3\ ,2[=$ ....

[7] Choose the correct answer:

(1) $2\sqrt{5}+3\sqrt{5}=$ ....

(a) $5\sqrt{10}$ (b) $5\sqrt{5}$ (c) $6\sqrt{5}$ (d) 30

(2) The multiplicative inverse of the number $\frac{\sqrt{3}}{6}$ is ....

(a) $\frac{\sqrt{6}}{3}$ (b) $2\sqrt{3}$ (c) $\frac{3}{\sqrt{6}}$ (d) $-\frac{\sqrt{3}}{6}$

(3) $\sqrt{3}+(-\sqrt{3})=$ ....

(a) $2\sqrt{3}$ (b) $2\sqrt{6}$ (c) $\sqrt{6}$ (d) zero

(4) 🕮 $-2\sqrt{3}\times\sqrt{3}=$ ....

(a) -6 (b) $-2\sqrt{3}$ (c) $2\sqrt{3}$ (d) 6

(5)	The additive inverse of the number $\frac{6}{\sqrt{2}}$ = ……… (a) $-2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $-3\sqrt{2}$ (d) $3\sqrt{2}$
(6)	The additive inverse of the number $(\sqrt{2}-\sqrt{5})$ = ……… (a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$
(7)	The multiplicative inverse of the number $\sqrt{5}$ is ……… (a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$
(8)	$(\sqrt{5}+3\sqrt{5})\div\sqrt{5}$ = ……… (a) $3\sqrt{5}$ (b) 3 (c) 5 (d) 4
(9)	If : $x=\sqrt{2}+10$, $y=\sqrt{2}-10$, then $(x+y)^2$ = ……… (a) 4 (b) 6 (c) 8 (d) $4\sqrt{2}$
(10)	$[2 , 5]-\{2 , 5\}$ = ……… (a) $[3 , 4]$ (b) $]2 , 5[$ (c) $\{2 , 5\}$ (d) $[2 , 5]$
(11)	If : $x^3+9=1$ where $x\in\mathbb{R}$, then x = ……… (a) -8 (b) -2 (c) 2 (d) 8
(12)	If : $x=\sqrt{3}+2$, then x^2 = ……… (a) 5 (b) 7 (c) $7+2\sqrt{3}$ (d) $7+4\sqrt{3}$
(13)	If : $x^2-y^2=60$, $x+y=5\sqrt{6}$, then = $x-y$ = ……… (a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$

[8] If $x=\sqrt{5}-2$ and $y=\sqrt{5}+2$, find the value of:

(1) $x+y$ = ……………………………………

(2) $x-y$ = ……………………………………

(3) xy = ……………………………………

[9] If $x = -\sqrt{3}$ and $y = 2\sqrt{3} - 3$, find the value of:

(1) $x + y$ = ...

(2) xy = ...

(3) $\frac{y}{x}$ = ...

Sheet (7)
Operations on the square roots

If a and b are two non negative real numbers , then

1 $\sqrt{a} \times \sqrt{b} = \sqrt{a\,b}$

For example :

- $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$
- $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

2 $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ where $b \neq 0$

For exomple :

- $\dfrac{\sqrt{8}}{\sqrt{2}} = \sqrt{\dfrac{8}{2}} = \sqrt{4} = 2$
- $\sqrt{\dfrac{16}{49}} = \dfrac{\sqrt{16}}{\sqrt{49}} = \dfrac{4}{7}$

3 $\dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{\sqrt{a}}{\sqrt{b}} \times \dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{\sqrt{ab}}{b}$ where $b \neq 0$

This operation is carried out to make the denominator an integer.

For example :

- $\dfrac{\sqrt{2}}{\sqrt{5}} = \dfrac{\sqrt{2}}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{10}}{5}$
- $\sqrt{\dfrac{3}{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}}{2}$

[1] Put each of the following in the form $a\sqrt{b}$ where a and b are two integers and b is the least possible value:

(1) $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

(2) $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$

(3) $2\sqrt{72} = 2 \times \sqrt{36 \times 2} = 2 \times 6\sqrt{2} = 12\sqrt{2}$

(4) $\frac{2}{5}\sqrt{1000} = \frac{2}{5}\sqrt{100 \times 10} = \frac{2}{5} \times 10\sqrt{10} = 4\sqrt{10}$

(5) $2\sqrt{\frac{1}{2}} = \sqrt{2^2 \times \frac{1}{2}} = \sqrt{2} \quad \left(x\sqrt{\frac{1}{x}} = \sqrt{x}\right)$

(6) $6\sqrt{\frac{2}{3}} = \sqrt{36 \times \frac{2}{3}} = 2\sqrt{6}$

[2] Simplify each of the following to the simplest form:

(1) $\sqrt{50} + \sqrt{8}$

= ....

= ....

(2) $3\sqrt{2} + \sqrt{8} - \sqrt{18}$

= ....

= ....

(3) $\sqrt{98} - \sqrt{128} - \sqrt{18} + 4\sqrt{2}$

= ....

= ....

(4) $\sqrt{27} + 5\sqrt{18} - \sqrt{300}$

= ....

= ....

(5) $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$

= ....

= ....

(6) $2\sqrt{5} + 4\sqrt{20} - 5\sqrt{\frac{1}{5}}$

= ....

= ....

(7) $2\sqrt{5} + 6\sqrt{\frac{1}{3}} - \sqrt{12} - 5\sqrt{\frac{1}{5}}$

= ....

= ....

(8) $\sqrt{18} - \frac{\sqrt{12}}{\sqrt{6}}$

= ....

= ....

(9) $2\sqrt{3} \times 5\sqrt{2}$

= ....

= ....

(10) $\sqrt{5} \times 2\sqrt{10}$

= ....

= ....

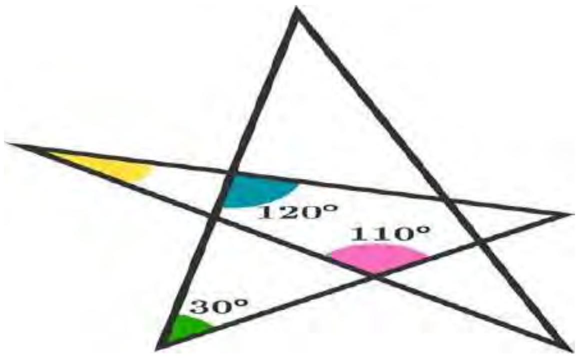
(11) $\sqrt{\frac{2}{7}} \times \sqrt{\frac{7}{2}}$ = ...

(12) $\frac{3\sqrt{15}}{\sqrt{5}}$ = ...

(13) $12\sqrt{\frac{2}{3}} \times \sqrt{54}$

= ....

= ....



[3] Simplify each of the following to the simplest form:

(1)	$\sqrt{6}(\sqrt{3}-\sqrt{2})$ = ...
(2)	$(3\sqrt{5}-\sqrt{7})(3\sqrt{5}+\sqrt{7})$ = ...
(3)	$(3\sqrt{2}-5)(3\sqrt{2}+5)$ = ...
(4)	$(\sqrt{2}+\sqrt{6})^2$ = ...
(5)	$(\sqrt{3}-\sqrt{2})^2$ = ...

[4] Choose the correct answer:

(1) $\frac{\sqrt{63}}{\sqrt{7}} = \dots\dots\dots$

(a) 3 (b) $\sqrt{3}$ (c) 9 (d) ± 3

(2) $\sqrt{8} - \sqrt{2} = \dots\dots\dots$

(a) $\sqrt{6}$ (b) $\sqrt{2}$ (c) 2 (d) 1

(3) $(\sqrt{8} + \sqrt{2})^2 = \dots\dots\dots$

(a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

(4) $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) = \dots\dots\dots$

(a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$

(5) $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} = \dots\dots\dots$

(a) 1 (b) $\sqrt{\frac{1}{4}}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

(6) $\frac{\sqrt{27}}{\sqrt{3}} \div \frac{\sqrt{72}}{\sqrt{2}} = \dots\dots\dots$

(a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 4

(7) The multiplicative inverse of the number $\sqrt{50}$ is $\dots\dots\dots$

(a) $\frac{\sqrt{2}}{10}$ (b) $\frac{-\sqrt{2}}{10}$ (c) $-5\sqrt{2}$ (d) $5\sqrt{2}$

(8) If : $x = \frac{\sqrt{6}}{\sqrt{2}}$, then $x^{-1} = \dots\dots\dots$

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $2\sqrt{3}$

(9) If : $x = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{28} + \sqrt{12}$, then $x =$ ………

(a) y (b) $\frac{1}{2}$ y (c) 2y (d) y^2

[5] If $x = \frac{\sqrt{50}-\sqrt{18}}{2}$ and $y = 2 - \sqrt{2}$, find in the simplest form:

……………………………………………………………………………………

(1) $x + y$ = ……………………………………………………………………………………

(2) xy = ……………………………………………………………………………………

Sheet (8)
The two conjugate numbers

If a and b are two positive rational numbers , then each of the two numbers $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ is conjugate to the other one and we find that

- Their sum $= (\sqrt{a}+\sqrt{b}) + (\sqrt{a}-\sqrt{b}) = 2\sqrt{a}$ = twice the first term.
- Their product $= (\sqrt{a}+\sqrt{b})\ (\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

The difference:

Greater – smaller = $(\sqrt{a}+\sqrt{b}) - (\sqrt{a}-\sqrt{b}) = 2\sqrt{b}$ **(twice the second term)**

Smaller – greater = $(\sqrt{a}-\sqrt{b}) - (\sqrt{a}+\sqrt{b}) = -2\sqrt{b}$ **(negative twice the second term)**

Example: $(\sqrt{2}+\sqrt{5})$ its conjugate is $(\sqrt{2}-\sqrt{5})$

Their sum 2 of 1st term	G - S 2 of 2nd term	S - G -2 of 2nd term	Their product $(1^{st})^2-(2^{nd})^2$
$2\sqrt{2}$	$2\sqrt{5}$	$-2\sqrt{5}$	$2-5=-3$

Exercise (1): $(3-\sqrt{7})$ its conjugate is ....

Their sum	G - S	S - G	Their product

Exercise (2): $(3\sqrt{5}-\sqrt{6})$ its conjugate is ....

Their sum	G - S	S - G	Their product

Remarks:

- $x^2 - y^2 = (x + y)(x - y)$
- $x^2 + 2xy + y^2 = (x + y)^2$
- $x^2 - 2xy + y^2 = (x - y)^2$

[1] Choose the correct answer:

(1)	The conjugate of $(\sqrt{3} - \sqrt{5})$ is ………… (a) $\sqrt{5} - 3$ (b) $\sqrt{3} + \sqrt{5}$ (c) $-\sqrt{3} - \sqrt{5}$ (d) $\sqrt{5} - \sqrt{3}$
(2)	$(\sqrt{5} + \sqrt{3})^2(\sqrt{5} - \sqrt{3})^2 =$……… (a) 4 (b) 2 (c) 8 (d) 3
(3)	The number $\frac{4}{3+\sqrt{5}}$ in the simplest form is ………… (a) $3 + \sqrt{5}$ (b) $3 - \sqrt{5}$ (c) $\sqrt{3} + \sqrt{5}$ (d) $3\sqrt{5}$
(4)	[2 , 5] - {2} = ………… (a)]2 , 5[(b)]2 , 5] (c) [2 , 5[(d) [2 , 5]
(5)	The conjugate of the number $\frac{1}{\sqrt{3}+\sqrt{2}}$ is ………… (a) $\sqrt{3} - \sqrt{2}$ (b) $\sqrt{3} + \sqrt{2}$ (c) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (d) $-\sqrt{3} - \sqrt{2}$

[2] If $x = \sqrt{5} + \sqrt{3}$ and $y = \frac{2}{\sqrt{5}+\sqrt{3}}$, prove that x and y are conjugate numbers then find the value of $x^2 + 2xy + y^2$.

…………………………………………………………

…………………………………………………………

…………………………………………………………

…………………………………………………………

[3] If $x=\sqrt{5}-\sqrt{2}$ and $y=\frac{3}{\sqrt{5}-\sqrt{2}}$, prove that x and y are conjugate numbers then find the value of $x^2-2xy+y^2$.

[4] If $a=\sqrt{3}+\sqrt{2}$ and $b=\frac{1}{\sqrt{3}+\sqrt{2}}$, find the value of a^2-b^2.

[5] If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$ and $y=\frac{4}{\sqrt{7}+\sqrt{3}}$, find the value of x^2y^2.

[6] If $x=\sqrt{5}+\sqrt{2}$ and $y=\sqrt{5}-\sqrt{2}$, find the value of $\frac{x+y}{xy-1}$.

[7] If $x = \frac{2}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{2}{\sqrt{5}+\sqrt{3}}$, find the value of $x^2 - xy + y^2$.

....

....

....

....

[8] If $x = \frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}}$ and $y = \frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$, find:

(1) $x^2 + y^2$ = ...

....

....

(2) xy = ...

....

....

(3) Prove that: $\frac{x^2+y^2}{xy} = 38$

....

....

....

....

[9] Complete:

(1)	$(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) = \cdots\cdots\cdots$
(2)	If : $x = 3+\sqrt{2}$, then its conjugate is $\cdots\cdots\cdots$ and the product of multiplying x by its conjugate is $\cdots\cdots\cdots$
(3)	The conjugate number of the number $\frac{1}{\sqrt{3}-\sqrt{2}}$ is $\cdots\cdots\cdots$
(4)	The conjugate number of the number $1+\frac{7}{\sqrt{7}}$ in the simplest form is $\cdots\cdots\cdots$
(5)	The multiplicative inverse for $(\sqrt{3}+\sqrt{2})$ in its simplest form is $\cdots\cdots\cdots$
(6)	If : $x = 2+\sqrt{5}$ and y is the conjugate number of x , then $(x-y)^2 = \cdots\cdots\cdots$
(7)	If : $\frac{x}{5-\sqrt{5}} = 5+\sqrt{5}$, then the value of x in its simplest form is $\cdots\cdots\cdots$
(8)	If : $\frac{1}{x} = \sqrt{5}-2$, then the value of x in its simplest form is $\cdots\cdots\cdots$
(9)	If : $x = \sqrt{3}+2$, $y = \sqrt{3}-2$, then $(xy , x+y)$ equals $\cdots\cdots\cdots$
(10)	$(\sqrt{2}+\sqrt{3})^{-9}(\sqrt{2}-\sqrt{3})^{-9} = \cdots\cdots\cdots$

Sheet (9)
Operations on the cube roots

If a and b are two real numbers , then

1 $\sqrt[3]{a}\times\sqrt[3]{b}=\sqrt[3]{ab}$

For example:

- $\sqrt[3]{3}\times\sqrt[3]{9}=\sqrt[3]{3\times 9}=\sqrt[3]{27}=3$
- $\sqrt[3]{2}\times\sqrt[3]{-4}=\sqrt[3]{2\times -4}=\sqrt[3]{-8}=-2$

2 $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}=\sqrt[3]{\frac{a}{b}}$ (where $b\neq 0$)

For example:

- $\frac{\sqrt[3]{32}}{\sqrt[3]{4}}=\sqrt[3]{\frac{32}{4}}=\sqrt[3]{8}=2$
- $\frac{\sqrt[3]{54}}{\sqrt[3]{-2}}=\sqrt[3]{\frac{54}{-2}}=\sqrt[3]{-27}=-3$

Remarks

* If a and b are two real numbers , then :

1 $\sqrt[3]{a^3+b^3}\neq a+b$, $\sqrt[3]{a^3-b^3}\neq a-b$

2 $\sqrt[3]{-a}=-\sqrt[3]{a}$

3 $a\sqrt[3]{b}=\sqrt[3]{a^3b}$

For example :
- $3\sqrt[3]{\frac{1}{9}}=\sqrt[3]{27\times\frac{1}{9}}=\sqrt[3]{3}$
- $8\sqrt[3]{\frac{1}{4}}=4\times 2\sqrt[3]{\frac{1}{4}}=4\sqrt[3]{8\times\frac{1}{4}}=4\sqrt[3]{2}$

4 $\sqrt[3]{\frac{a}{b}}=\sqrt[3]{\frac{a}{b}\times\frac{b^2}{b^2}}=\sqrt[3]{\frac{a\,b^2}{b^3}}=\frac{1}{b}\sqrt[3]{a\,b^2}$

For example :
- $\sqrt[3]{\frac{1}{3}}=\sqrt[3]{\frac{1}{3}\times\frac{9}{9}}=\sqrt[3]{\frac{9}{27}}=\frac{1}{3}\sqrt[3]{9}$

Important Remarks

$\sqrt[3]{16}=\sqrt[3]{8}\times\sqrt[3]{2}=2\sqrt[3]{2}$

$\sqrt[3]{24}=\sqrt[3]{8}\times\sqrt[3]{3}=2\sqrt[3]{3}$

$\sqrt[3]{54}=\sqrt[3]{27}\times\sqrt[3]{2}=3\sqrt[3]{2}$

$\sqrt[3]{81}=\sqrt[3]{27}\times\sqrt[3]{3}=3\sqrt[3]{3}$

$\sqrt[3]{128}=\sqrt[3]{64}\times\sqrt[3]{2}=4\sqrt[3]{2}$

$\sqrt[3]{40}=\sqrt[3]{8}\times\sqrt[3]{5}=2\sqrt[3]{5}$

$\sqrt[3]{250}=\sqrt[3]{125}\times\sqrt[3]{2}=5\sqrt[3]{2}$

$\sqrt[3]{135}=\sqrt[3]{27}\times\sqrt[3]{5}=3\sqrt[3]{5}$

[1] Find the result in its simplest form:

(1) $\sqrt[3]{2} \times \sqrt[3]{32}$ = ...

(2) $\dfrac{\sqrt[3]{72}}{\sqrt[3]{9}}$ = ...

(3) $\dfrac{4\sqrt[3]{-54}}{2\sqrt[3]{-2}}$ = ...

(4) $\frac{1}{2}\sqrt[3]{10} \times 6\sqrt[3]{100}$ = ...

(5) $\sqrt[3]{\frac{2}{5}} \times \sqrt[3]{\frac{4}{25}}$ = ...

(6) $\sqrt[3]{\frac{3}{4}} \div \sqrt[3]{\frac{2}{9}}$ = ...

[2] Find the result in its simplest form:

(1) $\sqrt[3]{16} - \sqrt[3]{2}$ = ...

= ....

(2) $\sqrt[3]{81} + \sqrt[3]{-24}$ = ...

= ....

(3) $2\sqrt[3]{54} - 5\sqrt[3]{2} + \sqrt[3]{16}$ = ...

= ....

(4) $\sqrt[3]{125}-\sqrt[3]{24}$

= ....

= ....

(5) $\sqrt[3]{54}+\sqrt[3]{16}-\sqrt[3]{250}$

= ....

= ....

(6) $\sqrt[3]{16}-\frac{1}{3}\sqrt[3]{54}+\sqrt[3]{-2}$

= ....

= ....

(7) $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}$

= ....

= ....

(8) $\sqrt[3]{54}\times\sqrt[3]{16}\div(\sqrt[3]{4}\times 6)$

= ....

= ....

[3] Simplify each of the following:

(1) $\frac{7}{3}\sqrt{18}+\sqrt[3]{54}-7\sqrt{2}+\sqrt[3]{16}$

= ....

= ....

(2) $\sqrt{27}+\frac{1}{3}\sqrt[3]{27}-9\sqrt{\frac{1}{3}}-1$

= ....

= ....

(3) $\sqrt[3]{-16}+\frac{14}{\sqrt{7}}-\sqrt{28}+\sqrt[3]{54}$

= ....

= ....

(4) $\sqrt{18}+\sqrt[3]{54}-\frac{\sqrt{216}}{\sqrt{12}}-\sqrt[3]{16}$

= ....

= ....

(5) $5\sqrt{2}-\frac{1}{2}\sqrt{200}+(\sqrt[3]{5}\times\sqrt[3]{25})$ = ...

= ....

[4]

If $a=\sqrt[3]{5}+1$, $b=\sqrt[3]{5}-1$ **Find the value of the following :**

1 $(a-b)^5$

= ....

= ....

2 $(a+b)^3$

= ....

= ....

[5]

If $x=3+\sqrt[3]{6}$, $y=3-\sqrt[3]{6}$ **Find the value of :** $\left(\frac{x-y}{x+y}\right)^3$

....

....

....

....

....

[6] Complete:

(1) $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{-12} = \dots\dots\dots$

(2) $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt{\dots\dots\dots}$

(3) $\sqrt[3]{54} - \sqrt[3]{-16} = \sqrt[3]{\dots\dots\dots}$

(4) The conjugate of the number $\frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$ is $\dots\dots\dots$

(5) $[0 , 5] - [0 , 3] = \dots\dots\dots$

(6) $\sqrt[3]{54} - \sqrt[3]{2} = \dots\dots\dots$

(7) If : $x = \sqrt[3]{2} + 1$ and $y = \sqrt[3]{2} - 1$, then $(x + y)^3 = \dots\dots\dots$

(8) $2\sqrt{\frac{1}{2}} - \sqrt{2} = \dots\dots\dots$

(9) The number $-\sqrt{11}$ is included between the two consecutive integers $\dots\dots\dots$ and $\dots\dots$

[7] Choose the correct answer:

(1) 📖 $\sqrt[3]{54} + \sqrt[3]{-2} = \dots\dots\dots$

(a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$

(2) 📖 $\sqrt[3]{-64} + \sqrt{16} = \dots\dots\dots$

(a) zero (b) 8 (c) -8 (d) ± 8

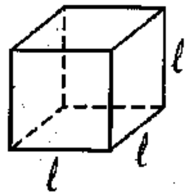
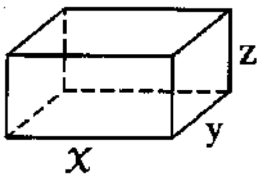
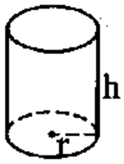
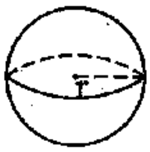
(3) $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \dots\dots\dots$

(a) 8 (b) -2 (c) 2 (d) $2\sqrt[3]{2}$

(4)	$\sqrt[3]{2}+\sqrt[3]{2}=$ .... (a) $\sqrt[3]{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[3]{8}$ (d) $\sqrt[3]{16}$
(5)	$\sqrt[3]{\frac{2}{9}}=$ .... (a) $\frac{\sqrt[3]{6}}{3}$ (b) $\sqrt[3]{\frac{1}{6}}$ (c) $\sqrt[3]{6}$ (d) $\sqrt[3]{2}$
(6)	If : $X =]-\infty , 0[$, then $\tilde{X} =$ .... (a) $\mathbb{R}_+$ (b) $[0 , \infty[$ (c) $]-\infty , 0]$ (d) $\mathbb{R}_-$
(7)	The multiplicative inverse of the number $\sqrt{\frac{3}{2}}$ is .... (a) $\frac{2}{3}\sqrt{2}$ (b) $\frac{3\sqrt{2}}{2}$ (c) $\frac{\sqrt{6}}{3}$ (d) $-\sqrt{\frac{3}{2}}$
(8)	The irrational number in the following numbers is .... (a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $2\sqrt{2}$
(9)	$]-1 , 3] \cap [-3 , -1] =$ .... (a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$
(10)	If : $x=\sqrt{3}+\sqrt{2}$ and $x\,y = 1$, then $y =$ .... (a) $\sqrt{2}-\sqrt{3}$ (b) $\sqrt{3}+\sqrt{2}$ (c) $\sqrt{3}-\sqrt{2}$ (d) 1

Sheet (10)
Applications on the real numbers

In the following , we will summarize the previous rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	$\pi r^2 h$
The sphere		–	$4\pi r^2$	$\frac{4}{3}\pi r^3$

THE CUBE

[1] A cube with volume 125 cm3. Find its total area and its lateral area.

....

....

[2] A cube whose lateral area is 36 cm2. Find its total area, and its volume.

....

....

....

[3] The perimeter of one face of a cube is 12 cm. Find its volume, and its lateral area.

....

....

....

[4] The sum of lengths of all edges of a cube is 60 cm. Find its volume and its total area.

....

....

....

[5] Complete:

(1)	If the edge length of a cube is 5 cm. , then its volume = ... cm^3.
(2)	The edge length of a cube is 4 cm. , then its total area = ... cm^2.
(3)	The lateral area of a cube whose edge length is ℓ cm. = ... cm^2.
(4)	The cube whose volume ℓ^3 cm^3. , its total area = ... cm^2.
(5)	The cube whose edge length is 2 ℓ , then its volume = ... cm^3.

[6] Choose the correct answer:

(1)	The volume of a cube is 1 cm^3 , then the sum of its edge lengths = ... cm. (a) 1 (b) 6 (c) 8 (d) 12
(2)	📖 The volume of a cube is 64 cm^3 , then its lateral area = ... cm^2 (a) 4 (b) 8 (c) 64 (d) 96
(3)	If the total area of a cube is 96 cm^2 , then the area of one face = ... cm^2 (a) 16 (b) 64 (c) 24 (d) 48
(4)	If the area of the six faces of a cube = 54 cm^2 , then its volume = ... cm^3 (a) 54 (b) 44 (c) 72 (d) 27
(5)	If the volume of a cube = 64 cm^3 , then the length of a diagonal of one face = ... cm. (a) 16 (b) $4\sqrt{2}$ (c) 32 (d) 64
(6)	The volume of a cube is 5 cm^3 If the edge length became twice the first , then its volume = ... cm^3 (a) 10 (b) 20 (c) 30 (d) 40
(7)	📖 The edge length of a cube whose volume is $2\sqrt{2}$ cm^3 = ... cm. (a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

THE CUBOID

[1] The dimensions of the base of a cuboid are 9 cm and 10 cm and its height is 5 cm. Find its volume, its lateral area and its total area.

....

....

....

[2] The height of a cuboid is 4 cm and its base is a square of side length 5 cm. Find its volume, its lateral area and its total area.

....

....

....

[3] The lateral area of a cuboid is 480 cm2 and its base is in the shape of a square whose side length is 10 cm. Calculate its height.

....

....

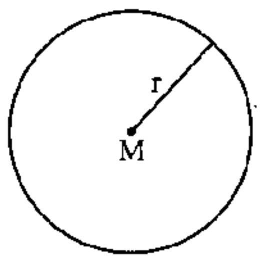
....

THE CIRCLE

If M is a circle with radius length r , then :

1 The circumference of the circle = $2 \pi r$ length unit.

2 The area of the circle = πr^2 square unit.



[1] A circle is of radius length 10.5 cm. Find each of its circumference and its area. $\left(\pi = \frac{22}{7}\right)$

....

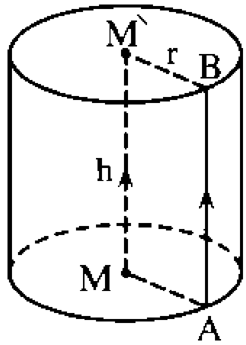
....

....

[2] The area of a circle is 25π cm^2. Calculate its circumference in terms of π.

[3] The area of a circle is 154 cm^2. Find its circumference and its diameter length. $\left(\pi = \frac{22}{7}\right)$

THE RIGHT CIRCULAR CYLINDER



1. The lateral area of the cylinder = $2\pi r h$ square unit.
2. The total area of the cylinder = the lateral area of the cylinder + twice the area of the base $= 2\pi r h + 2\pi r^2$ square unit.
3. The volume of the cylinder = the area of the base × height = $\pi r^2 h$ cube unit.

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

(1) A right circular cylinder , the radius length of its base is 14 cm. and its height is 20 cm. Find the volume and the total area of the cylinder. « 12320 cm^3, 2992 cm^2 »

(2) Find the lateral area for a right circular cylinder of volume 924 cm^3, and of a height 6 cm.

« 264 cm^2 »

(3) 📖 Find the total area of a right circular cylinder of volume 7536 cm^3 and its height is 24 cm. ($\pi = 3.14$)

« 2135.2 cm^2 »

(4) 📖 Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 72π cm^3

« $2\sqrt[3]{9}$ cm. »

THE SPHERE

1 The area of the sphere = $4\pi r^2$ square unit.

2 The volume of the sphere = $\frac{4}{3}\pi r^3$ cube unit.

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

(1) Find the volume and the surface area of a sphere if the length of its diameter is 4.2 cm.

« 38.808 cm^3 , 55.44 cm^2 »

(2) The volume of a sphere is 4188 cm^3. Find its radius length. ($\pi = 3.141$) « 10 cm. »

....

....

(3) **The volume of a sphere = $\frac{500}{3}\pi$ $cm.^3$ Find the length of its diameter.**

....

....

(4) The volume of a sphere is 562.5π cm^3.

Find its surface area in terms of π « 225π »

....

....

General Revision on Applications on the Real Numbers

[1] Complete:

(1) The sphere whose volume = 36π cm^3 has a radius length = ... cm.

(2) A right circular cylinder, its volume is 343π cm^3. If its height equals its base radius length, then its height = ... cm.

(3) The volume of a cube is 64 cm^3, then its total area is .... cm^3.

[2] Choose the correct answer:

(1) The circle whose radius length = $\sqrt{14}$ cm. has an area = ... cm^2.

(a) 14π (b) $2\sqrt{14}\pi$ (c) 14 (d) $2\sqrt{14}$

(2) $]2, 4[\cup \{2, 4\}$ = ...

(a) $\varnothing$ (b) $\{2, 4\}$ (c) $[2, 4]$ (d) $]2, 4[$

(3) The right circular cylinder whose base radius length = 3 cm. and its height = 5 cm. its volume = ……… cm^3

(a) 15π (b) 75π (c) 45π (d) $\frac{3}{5}\pi$

(4) If : $-\sqrt{25} = \sqrt[3]{x}$, then x = ………

(a) -5 (b) -25 (c) -125 (d) 125

(5) 📖 The volume of the sphere whose diameter length is 6 cm. = ……… cm^3

(a) 288 (b) 12π (c) 36π (d) 288π

(6) If the volume of a sphere = $\frac{9}{16}\pi$ cm^3 , then its radius length = ……… cm.

(a) 3 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

(7) If the surface area of a sphere is 9π cm^2 , then its diameter length = ……… cm.

(a) 9 (b) 3 (c) 1.5 (d) 6

(8) If three quarters of volume of a sphere equals 8π cm^3 , then the length of its radius equals ……… cm.

(a) 64 (b) 8 (c) 4 (d) 2

Sheet (11)
Solving equations and and inequalities of the first degree in one variable in R

[1] Find the solution set for each of the following equations in R, then graph the solution on the number line:

(1) $x + 5 = 0$

....

....

(2) $5x + 6 = 1$

....

....

(3) $2x + 4 = 3$

....

....

(4) $2x - 3 = 4$

....

....

(5) $\sqrt{5}x - 1 = 4$

....

....

(6) $x - 1 = \sqrt{3}$

....

....

(7) $\sqrt{3}x - 1 = 2$

....

....

(8) $7x - \sqrt{7} = 6\sqrt{7}$

....

....

(9) $x - \sqrt{5} = 1$

(10) $x - \sqrt{3} = 1$

[2] **Find the solution set for each of the following inequalities in R in the form of interval, then graph the solution on the number line:**

(1) $2x > 6$

(2) $-7x \geq -14$

(3) $x + 3 \leq 5$

(4) $5 - x > 3$

(5) $2x + 5 \geq 3$

(6) $1 - 5x < 6$

(7) $\frac{1}{2}x + 1 \leq 2$

(8) $3 - 2x \leq 7$

(9) $3 < x + 2 \le 6$

(10) $-5 < x + 3 < 9$

(11) $-3 \le -x \le 3$

(12) $1 < 5 - x \le 3$

(13) $\sqrt[3]{-8} \le x + 1 \le \sqrt{9}$

(14) $5 < 3 - x \le 3^2$

(15) $|-3| < 2x - 1 < 5$

(16) $3x < 2x + 4$

(17) $7x - 9 \ge 4x$

(18) $5x - 3 < 2x + 9$

(19) $x + 3 \ge 2x \ge x - 2$

(20) $4x \le 5x + 2 \le 4x + 3$

(21) $x - 1 < 3x - 1 \leq x + 1$

(22) $2x - 1 < 3x - 2 \leq 2x + 1$

(23) $1 \leq 3 - 2x < 5$

(24) $2x - 1 \geq 7$

(25) $-7 \leq 3x + 2 < 11$

(26) $x + 1 \leq 2x - 3 < x + 4$

(27) $-2 < 3x + 7 \leq 10$

(28) $-3 < 2x - 3 \leq \sqrt[3]{125}$

(29) $3 \leq 2x - 1 < 11$

(30) $6 < 2x + 4 \leq 10$

(31) $5 < 2x - 3 \leq 11$

(32) $2x + 4 \geq 3x - 3 > 2x + 1$

[1] Complete:

(1)	If $x - 3 \geq 0$, then x ....
(2)	If $5x < 15$, then x ....
(3)	If $1 - x > 4$, then x ....
(4)	If $-2x \leq 3$, then x ....
(5)	If $\sqrt{2}\, x \leq 4$, then x ....
(6)	The S.S. of the inequality $-5 \leq -x < 2$ in $\mathbb{R}$ is ....
(7)	If $-3 < x < 3$ where $x \in \mathbb{R}$, then $2x \in]-6 , ...[$

[2] Choose the correct answer:

(1) The S.S. of the inequality : $x + 3 < 3$ in $\mathbb{R}$ is ....

(a) $]-\infty , 0[$ (b) $]-\infty , 0]$ (c) $[0 , \infty[$ (d) $]0 , \infty[$

(2) The S.S. of the inequality : $1 > x - 5 > -1$ in $\mathbb{R}$ is ....

(a) $[4 , 6]$ (b) $]4 , 6[$ (c) $]4 , 6]$ (d) $[4 , 6[$

(3) If $x > 5$, then $-x$ ....

(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

(4) If $-2 < x < 2$, then $2x + 3$ belongs to ....

(a) $[-1 , 7]$ (b) $]-1 , 5[$ (c) $]-1 , 7[$ (d) $]-4 , 6[$

Sheet (12)
Relation between two variables

(1) **Complete the following ordered pairs which satisfy the relation : $y = 3x - 1$**

(5 , ……….) , (2 , ……….) , (0 , ……….) , (– 3 , ……….)

(2) **Show which of the following ordered pairs satisfy the relation : $y - 4x = 7$**

1. (1 , 2)
2. (3 , – 5)
3. (– 1 , 3)

(3) Find four ordered pairs satisfying the relation $y = 2x - 1$ and represent it. ……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

(4) Find three ordered pairs that satisfy the relation $x + 2y = 6$, then represent it graphically.

(5) Graph the relation $y = 2x - 3$

(6) Represent graphically the relation $x + y = 2$

....

....

(7) Represent the relation $2y - x = 2$ graphically

....

....

[8] Complete:

(1) If (2,3) satisfies the relation $x + y = k$, then k = ...

(2) If (k,2k) satisfies the relation $x + y - 15 = 0$, then k = ...

(3) If (-2,k) satisfies the relation $2x + 3y = 35$, then k = ...

[9] Choose the correct answer:

(1) If (2 , – 5) satisfies the relation : $3x - y + c = 0$, then c = ...

(a) 1 (b) – 1 (c) 11 (d) – 11

(2) Which of the following ordered pairs satisfies the relation : $2x + y = 5$?

(a) (– 1 , 3) (b) (1 , 3) (c) (3 , 1) (d) (2 , 2)

(3) (3 , 2) does not satisfy the relation ....

(a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$

(4) The relation : $3x + 8y = 24$ is represented by a straight line intersecting y-axis at the point ....

(a) (0 , 8) (b) (8 , 0) (c) (0 , 3) (d) (3 , 0)

(5) The relation $2x + 7y = 14$ is represented by a straight line intersecting x-axis at the point ....

(a) (2 , 0) (b) (0 , 2) (c) (7 , 0) (d) (0 , 7)

(6) If : (2 k , 3 k) satisfies the relation $x + y = 15$, then k = ...

(a) 5 (b) 3 (c) – 5 (d) – 3

Sheet (13)
Slope of straight line

If a point moves on a straight line L from the location A (x_1, y_1) to the location B (x_2, y_2), then :

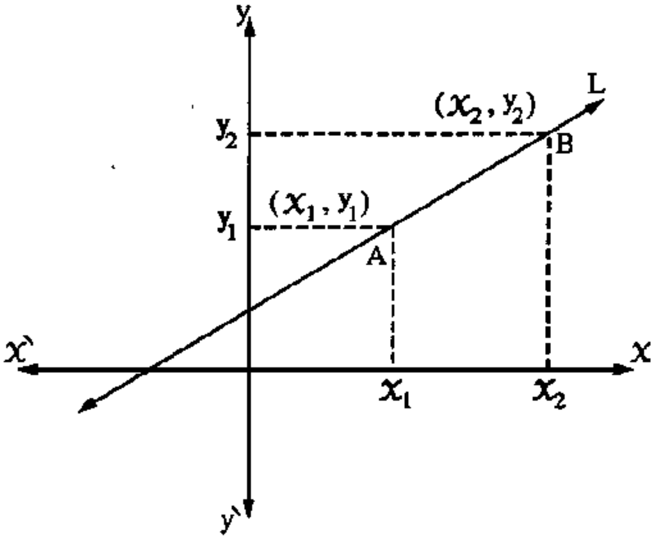
The change in the x-coordinates $= x_2 - x_1$

It is called (the horizontal change).

The change in the y-coordinates $= y_2 - y_1$

It is called (the vertical change).

The ratio of the change in the y-coordinates to the change in the x-coordinates is called the slope of the straight line (m).



$$\text{The slope of the straight line} = \frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$$

i.e. $S = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$, S is undefined if $x_1 = x_2$

1 **Classify the slope of the straight line in each of the following figures showing whether it is (positive – negative – zero – undefined)**

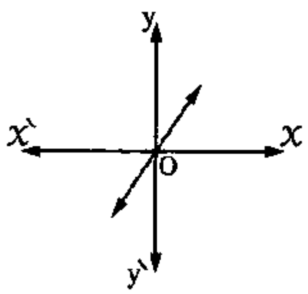


Fig. (1)

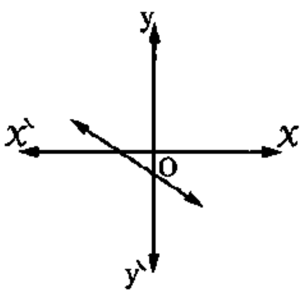


Fig. (2)

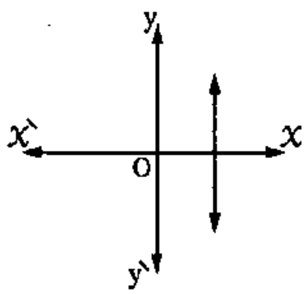


Fig. (3)

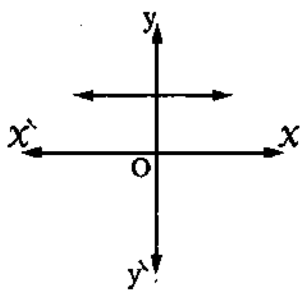


Fig. (4)

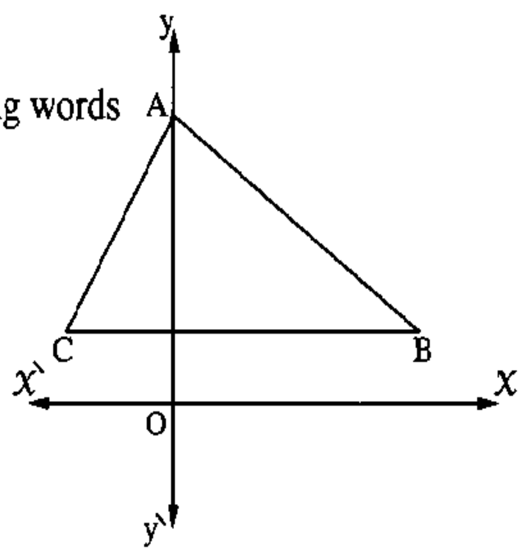
[2] Find the slope of the straight line passing through the two points in each of the following:

(1)	A (1 , 3) , B (3 , 4)
(2)	A (1 , 2) , B (5 , 0)
(3)	A (2 , – 1) , B (4 , – 1)
(4)	A (5 , 2) , B (5 , 4)
(5)	A (3,5) , B (5,-1)
(6)	A (-1,3) , B (2,4)
(7)	A (1,3) , B (2,1)
(8)	A (1,3) , B (2,3)

[3]

In the opposite figure :

ABC is a triangle. Complete by using one of the following words (positive , negative , zero , undefined)



1. The slope of $\overleftrightarrow{AB}$ is ....
2. The slope of $\overleftrightarrow{BC}$ is ....
3. The slope of $\overleftrightarrow{AO}$ is ....
4. The slope of $\overleftrightarrow{AC}$ is ....

[4] Complete:

(1)	The slope of any horizontal straight line equals ……….
(2)	The slope of any straight line parallel to y-axis is ……….
(3)	The straight line whose slope = zero is parallel to ……….
(4)	If A , B and C are collinear then the slope of $\overleftrightarrow{AB}$ = the slope of ……….
(5)	The slope of the straight line $\overleftrightarrow{AB}$ where A (2 , 3) and B (0 , 4) is ……….
(6)	If the slope of the straight line which passes through the two points (1 , 3) , (3 , k) equals 3 , find the value of k «9»
(7)	If the slope of the straight line which passes through the two points (3 , c) and (5 , – 2) equals – 3 , find the value of c «4»

[4] Prove that the points A(2,5) , B(0,1) and C(5,12) are collinear.

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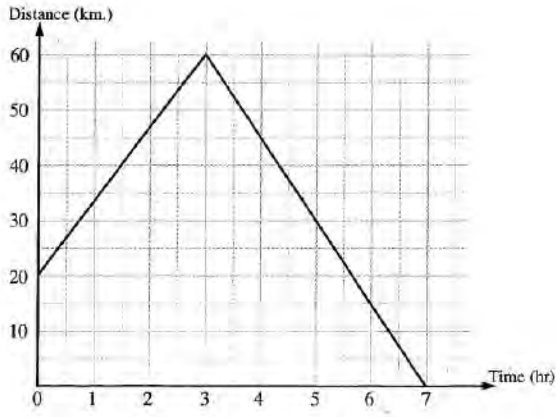
Real life applications on the slope

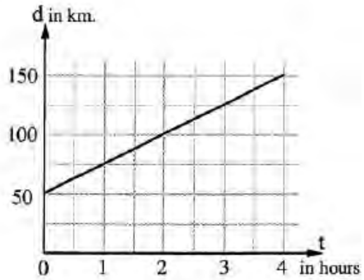
1 An irrigation machine consumes 2.47 litres of diesel to work for 3 hours. If the machine works for 10 hours , how many litres of diesel will the machine consume ? « $8\frac{7}{30}$ litres »

2 A car moves with uniform velocity such that it covers 180 km. per 3 hours. If the car moves for 5 hours , what is the covered distance ? « 300 km. »

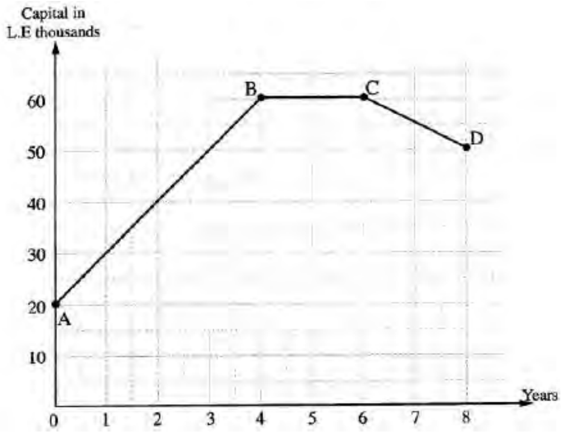
3 The opposite figure represents the motion of a bicycle measured from a constant point.
Find the regular speed of the bicycle during :
(1) The first three hours.
(2) The next four hours.
Find the total distance covered by the bicycle.



4 The opposite graph represents the motion of a car moving with uniform velocity.
Determine the velocity of the car.



5 📖 **The opposite figure shows the capital change of a company during 8 years :**



(1) Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

What is the meaning of each ?

(2) Find the starting capital of the company.

« 10 , 0 , – 5 , 20 thousand pounds »

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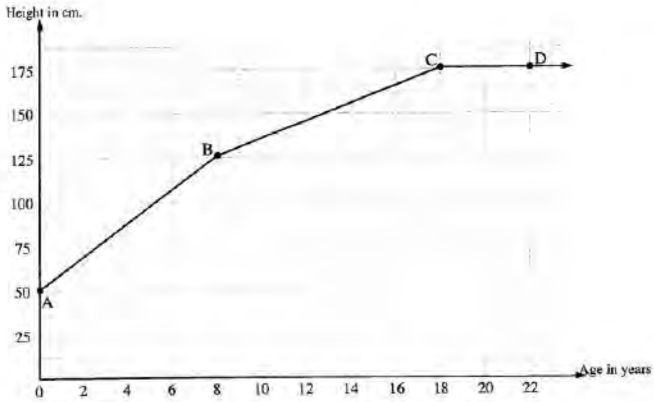
6 📖 **The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :**



(1) Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

What is the meaning of each ?

(2) Calculate the difference between the height of this person when he was 8 years old and his height when he was 30 years old.

« $9\frac{3}{8}$, 5 , 0 , 50 cm. »

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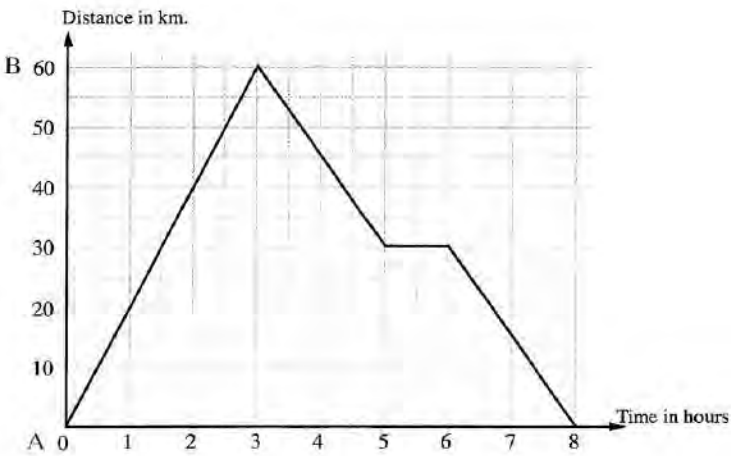
7 **The opposite graph shows the relation between the distance in km. and the time (t) in hours for a bicycle which moved between two towns A and B going and returning back. Answer the following :**



(1) What is the uniform velocity during the going trip?

(2) What is the average velocity during returning back ?

(3) What is the meaning of the horizontal line segment in the graph ?

« 20 km./hr. , 12 km./hr. »

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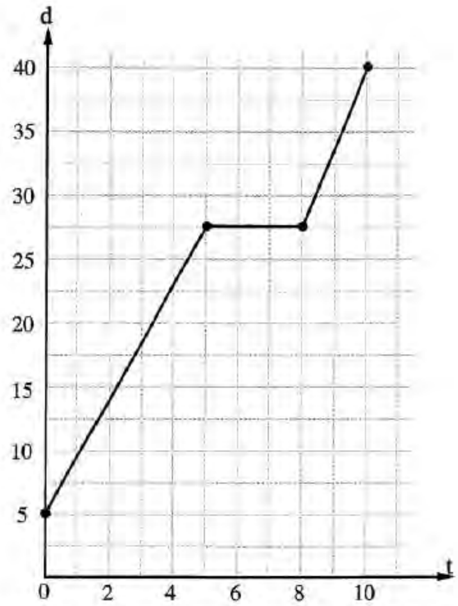
8 **A farmer wanted to complete digging a well in his farm. He rented a digging machine. The opposite graph shows the depth of the well (d) in metres after time (t) in hours , find :**



(1) The depth of the well before beginning digging.

(2) The depth of the well after finishing digging.

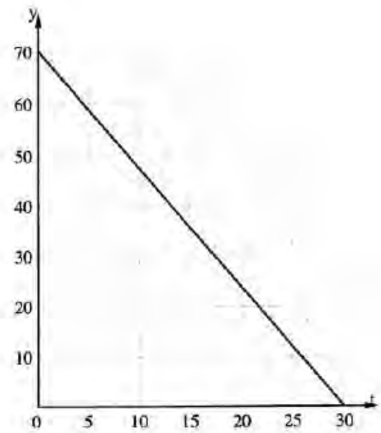
(3) The total time which the machine took in digging the well.

(4) The average of depth of the well which the machine digs within the first five hours.

(5) The average of the depth of the well within the last two hours of digging.

« 5 m. , 40 m. , 10 hr. , 4.5 m./hr. , 6.25 m./hr. »

9 **Magdi filled the tank of his car by fuel. The opposite figure represents the relation between the time (t) in hours and the amount of remained fuel in the tank (y) in litres :**



(1) What is the greatest capacity of the tank ?

(2) When will the tank become empty ?

(3) What is the amount of remained fuel after 15 hours ?

(4) What is the range of consumption of fuel in each hour ? « 70 L , 30 hr. , 35 L , $2\frac{1}{3}$ L /hr. »

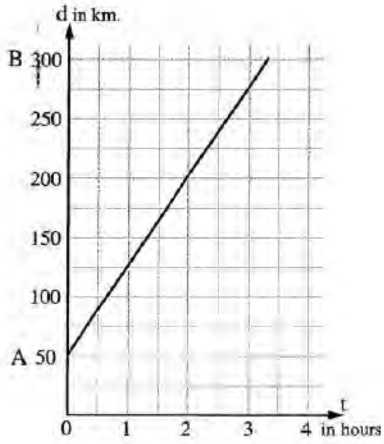
10 Bassim drove his car from the city A to the city B

The opposite graph shows the relation between the distance d in km. and the time t in hours.

Answer the following :

(1) What is the uniform velocity of the car of Bassim ?

(2) Find the distance between the car and the point 0 after three hours from the moment of beginning.



« 75 km./hr. , 275 km. »

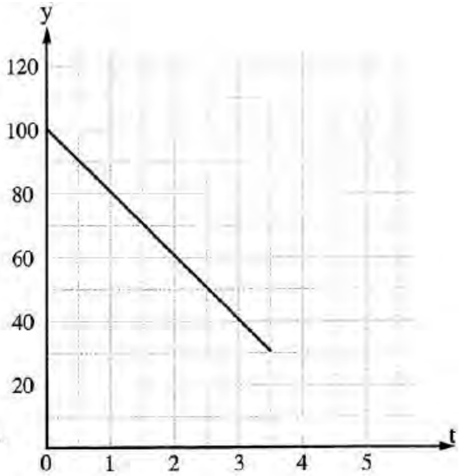
11 **A person read a book.**

The opposite graph shows the relation between the time (t) in hours and the number of remained pages (y) :

(1) How many pages are remained in the beginning ?

(2) Find the rate of reading pages per hour.

(3) When does this person finish reading this book ?



« 100 pages , 20 page/hr. , after 5 hours »

1-6 Operations on the real numbers

The properties of addition operation of real numbers

- **Closure :**

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

- **Commutative property :**

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

- **Associative property :**

  For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

- **The additive neutral :**

  For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

- **The additive inverse of every real number :**

  For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

- Subtraction operation is not commutative and it is not associative.

Write the additive inverse for every of the following numbers :

1) $\sqrt{2}$ ....

2) $-\sqrt[3]{5}$ ....

3) $\sqrt{2}+\sqrt{7}$ ....

4) $\sqrt[3]{5}-3$ ....

5) $-\sqrt{6}-\sqrt[3]{7}$ ....

Simplify to the simplest form :

6) $-3\sqrt{5}+4\sqrt{5}+(-2\sqrt{5})$

7) $4+\sqrt{3}-7-\sqrt{3}$

8) $2+2\sqrt{7}-1-5\sqrt{7}$

9) $3\sqrt{5}+\sqrt{3}-3\sqrt{5}+5\sqrt{3}$

The properties of multiplication operation of real numbers

- **Closure :**

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$
- **Commutative property :**

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$
- **The associative property :**

  For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$
- **The multiplicative neutral :**

  For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$
- **The multiplicative inverse of any non-zero real number :**

  For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$
- The division operation in $\mathbb{R}$ is not commutative and it is not associative.
- **Distributing multiplication on addition and subtraction**

$a(b \pm c) = ab \pm ac$

Find the result of each of the following :

10) $-2 \times 3\sqrt{5}$

11) $4\sqrt{2} \times \sqrt{2}$

12) $-2\sqrt{7} \times 4\sqrt{7}$

13) $\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}} \div \frac{1}{3\sqrt{2}}$

14) $\sqrt{5} \times \frac{1}{\sqrt{5}} \times \sqrt{5}$

15) $\frac{\sqrt{3}}{3} \times \frac{4\sqrt{5}}{20} \times \frac{5\sqrt{3}}{\sqrt{5}}$

Write each of the following such that the denominator is an integer :

16) $\frac{9}{\sqrt{3}}$

17) $-\frac{3}{\sqrt{2}}$

18) $\frac{5}{3\sqrt{5}}$

19) $\frac{3}{\sqrt{7}}$

20) $\frac{9}{2\sqrt{6}}$

Find each of the following :

21) $2\sqrt{3}(5\sqrt{3}-4)$

22) $5\sqrt{2}(3\sqrt{2}-2)$

23) $(7\sqrt{2}-5)(7\sqrt{2}+5)$

24) $(2\sqrt{3}-3)(2\sqrt{3}+3)$

25) $(2+\sqrt{3})(\sqrt{3}+7)$

26) $(5\sqrt{3}-2)^2$

Find the value of the expression :

27) $x^2+2xy+y^2$, If $x=5\sqrt{3}-2$, $y=5\sqrt{3}+2$

28) $x^2-2xy+y^2$, If $x=2\sqrt{3}-1$ and $y=2\sqrt{3}+1$

1-7 Operations on the square roots

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ where $b \neq 0$
- $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ where $b \neq 0$
- $a\sqrt{b} = \sqrt{a^2 b}$
- $\sqrt{a^2 + b^2} \neq a + b$
- $\sqrt{a^2 - b^2} \neq a - b$

Write each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value.

1) $\sqrt{27}$

....

....

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2) $5\sqrt{54}$

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3) $3\sqrt{\frac{2}{3}}$

....

....

....

4) $\frac{\sqrt{84}}{\sqrt{7}}$

....

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....

Simplify to the simplest form :

5) $\sqrt{45} - 2\sqrt{20} + 2\sqrt{5}$

6) $2\sqrt{18} + \sqrt{50} - 42\sqrt{\frac{1}{2}}$

7) $2\sqrt{27} - 3\sqrt{\frac{1}{3}} - \frac{6}{\sqrt{3}}$

8) $\sqrt{75} - 2\sqrt{27} + \sqrt{3}$

9) $2\sqrt{50} - 3\sqrt{2} - 4\sqrt{\frac{1}{8}}$

Find each of the following :

10) $2\sqrt{3}(\sqrt{6} + 5)$

11) $(3\sqrt{2} - 5)(3\sqrt{2} + 5)$

12) $(\sqrt{2}+\sqrt{6})^2$

find the value of

13) $a^2+2\sqrt{3}$, If $a=\frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$

Write each of the following such that the denominator is an integer :

14) $\frac{5\sqrt{3}}{2\sqrt{5}}$

15) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

1-8 the two conjugate numbers

- $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ is conjugate to the other
- Their sum $= (\sqrt{a}+\sqrt{b}) + (\sqrt{a}-\sqrt{b}) = 2\sqrt{a}$ = twice the first term.
- Their product $= (\sqrt{a}+\sqrt{b})\ (\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
- The product of the two conjugate numbers is always a rational number.

Important Remarks

- $(x-y)(x+y) = x^2 - y^2$
- $(x+y)^2 = x^2 + 2xy + y^2$
- $x^2 + xy + y^2 = (x+y)^2 - xy$
- $x^2 + y^2 = (x+y)^2 - 2xy$
- $(x-y)^2 = x^2 - 2xy + y^2$
- $x^2 - xy + y^2 = (x-y)^2 + xy$
- $x^2 + y^2 = (x-y)^2 + 2xy$

Complete the following :

1) $(\sqrt{2}+\sqrt{5})$ its conjugate is .... , their sum = ... , their product = ...
2) $(3-\sqrt{7})$ its conjugate is .... , their sum = ... , their product = ...
3) $(3\sqrt{5}-\sqrt{6})$ its conjugate is .... , their sum = ... , their product = ...

Write each of the following such that the denominator is a rational number.

4) $\frac{4}{\sqrt{7}-\sqrt{3}}$

5) $\frac{12}{\sqrt{6}-\sqrt{2}}$

6) $\frac{\sqrt{8}}{3+2\sqrt{2}}$

7) If $x = \frac{4}{2-\sqrt{2}}$ and $y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$,

write each of x and y such that its denominator is a rational number , then find $x + y$

8) If $x = \frac{2}{\sqrt{5}-\sqrt{3}}$ and $y = \sqrt{5}-\sqrt{3}$,

prove that x and y are conjugate numbers , then find the value of the expression :

a. $x^2 + 2xy + y^2$

b. $x^2 + xy + y^2$

9) If $x = \frac{3}{2\sqrt{2}-\sqrt{5}}$ and $y = 2\sqrt{2}-\sqrt{5}$, find the value of the expression : $x^2 - y^2$

1-9 Operations on the cube roots

If a and b are two real numbers , then

- $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$
- $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)
- $\sqrt[3]{a^3 + b^3} \neq a + b$
- $\sqrt[3]{a^3 - b^3} \neq a - b$
- $\sqrt[3]{-a} = -\sqrt[3]{a}$
- $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$
- $\sqrt[3]{\dfrac{a}{b}} = \sqrt[3]{\dfrac{a}{b} \times \dfrac{b^2}{b^2}} = \sqrt[3]{\dfrac{a b^2}{b^3}} = \dfrac{1}{b}\sqrt[3]{a b^2}$

Simplify each of the following in the simplest form :

1) $\sqrt[3]{\dfrac{2}{3}} \times \sqrt[3]{\dfrac{4}{9}}$

2) $\sqrt[3]{\dfrac{5}{4}} \div \sqrt[3]{\dfrac{2}{25}}$

3) $\sqrt[3]{24}+\sqrt[3]{3}-\sqrt[3]{81}$

4) $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}$

5) $\sqrt[3]{81}+\sqrt{12}-2\sqrt[3]{3}-2\sqrt{3}$

6) $2\sqrt[3]{4}\,(5\sqrt[3]{\frac{1}{2}}-\sqrt[3]{32})$

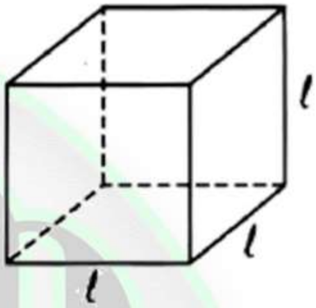
7) $5\sqrt[3]{2}-\sqrt{16}+\sqrt[3]{-54}$

8) $\sqrt[3]{72}+3\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

9) If $x=\sqrt[3]{5}+2$ and $y=\sqrt[3]{5}-2$, find the value of $(x+y)^3-(x-y)^3$

1-10 Applications on the real numbers

The Cube



- The area of each face = ℓ^2 square unit.
- Its lateral area = $4\ \ell^2$ square unit.
- Its total area = (the area of its 6 faces) = $6\ \ell^2$ square unit.
- Its volume = ℓ^3 cube unit.

find

1) **A cube with volume 125 $cm.^3$, find its total area and its lateral area.**

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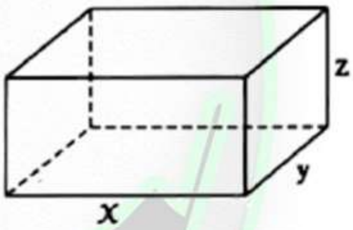
Complete the following table

2)

	Edge length of the cube	Area of one face	lateral area	Total area	Volume
(1)	3 cm.	....	....	....	....
(2)	....	49 cm^2.	....	....	....
(3)	....	....	144 cm^2.	....	....
(4)	....	....	....	150 cm^2.	....
(5)	....	....	....	....	64 cm^3.

The Cuboid

- Its lateral area = the perimeter of the base × height = $2(x + y) \times z$ square units.
- Its total area (the area of its six faces)
  = the lateral area + twice the area of the base
  = $2(x + y) \times z + 2xy = 2(xy + yz + zx)$ square units.



- Its volume = the area of the base × the height = $x \times y \times z$ cube unit.

3) **The height of a cuboid is 4 cm. and its base is a square of side length 5 cm. Find**

a. its volume b. its lateral area c. its total area.

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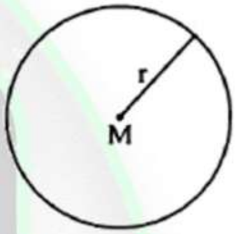
4) The dimensions of a cuboid are 3 cm. , 4 cm. and 5 cm. Calculate its volume and its total area.

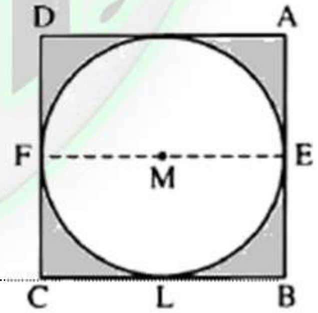
The Circle



- The circumference of the circle = $2\pi r$ length unit.
- The area of the circle = πr^2 square unit.

5) **The area of a circle is 25π cm^2. Calculate its circumference in terms of π**

6) **In the opposite figure : a circle M is drawn inside a square (touching its sides). If the area of the square = 196 cm^2 . , find :**

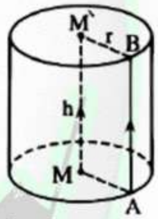


a. The area of the shaded part.

b. The perimeter of the shaded part.

7) The circumference of a circle is 88 cm. Find its area.

The right circular cylinder



- The lateral area of the cylinder = $2\pi r h$ square unit.
- The total area of the cylinder = the lateral area of the cylinder + twice the area of the base $= 2\pi r h + 2\pi r^2$ square unit.
- The volume of the cylinder = the area of the base × height = $\pi r^2 h$ cube unit.

8) **A right circular cylinder is of height 10 cm. and its volume is 1540 cm^3. Find its total area $(\pi = \frac{22}{7})$**

9) A right circular cylinder is of volume $90\,\pi$ cm.^{3} and its height is 10 cm. Find the diameter length of its base.

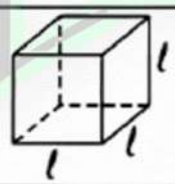
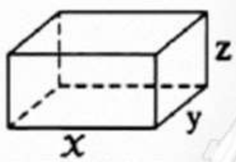
The Sphere

- The area of the sphere = $4\,\pi\, r^2$ square unit.
- The volume of the sphere = $\frac{4}{3}\,\pi\, r^3$ cube unit.

10) **The volume of a sphere = $\frac{500}{3}\,\pi$ cm.^{3} Find the length of its diameter.**

11) **A right circular cylinder is of height 6 cm. and its volume = $\frac{2}{3}$ the volume of a sphere whose radius length is 3 cm. Find the radius length of the base of the cylinder.**

12) The area of a sphere is 36π cm^2. Find its volume in terms of π

The solid		The lateral area	Total area	The volume
The cube		$4l^2$	$6l^2$	l^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	πr^2h
The sphere			$4\pi r^2$	$\frac{4}{3}\pi r^3$

1-11 Solving equations and inequalities of the 1st degree

Solving equations of the first degree in $\mathbb{R}$

Find in $\mathbb{R}$ the S.S of each of the following equations , then represent the solution on the number line.

1) $3x + 2 = 1$

2) $7x - \sqrt{7} = 6\sqrt{7}$

3) $\sqrt{3}x - 1 = 2$

4) $x - \sqrt{5} = 1$

5) $2x + 5 = 4$

6) $\sqrt{5}x - 1 = 4$

7) $x - \sqrt{3} = 2$

Solving the inequalities of the first degree in one unknown in $\mathbb{R}$

Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line.

8) $2x + 6 < 2$

9) $5 - 4x \leq -3$

10) $3x - 1 > 8$

11) $2 - 2x > -6$

12) $-3 < 2x - 1 \leq 5$

13) $3 < 3 - 5x < 13$

14) $-16 < 5x + 4 \le 9$

15) $x - 2 \ge 3x - 5$

16) $x - 1 < 3x - 3 \le x + 5$

17) $2x + 1 > 4x - 3 > 2x - 11$

2-1 Relation between two variables

1) What is the different possibilities for a person to pay L.E. 45 using two kinds of bills (banknotes) of L.E. 5 and L.E. 10

2) Find the different possibilities for a person to pay L.E. 65 of bills (banknotes) of L.E. 5 and L.E. 20

Find three ordered pairs satisfying each of the following relations :

3) $3x + y = 5$

4) $3x - 2y = 6$

5) $3x + y = 2$

6) $2x = 3$

7) $y = -2$

Find :

8) **Show which of the following ordered pairs satisfies the relation $2x - y = 1$:**

$(0, 1), (5, 3), (3, 5), (-2, -5)$

9) If $(-2, 1)$ satisfies the relation : $3x + by = 1$ Find the value of b

10) If $(k, 2k)$ satisfies the relation : $5x - y = 6$ Find the value of k

11) **If $(3k, 2k)$ satisfies the relation : $x - 3y = 9$, find the value of k**

Represent the relation graphically :

12) $2x - y = 3$

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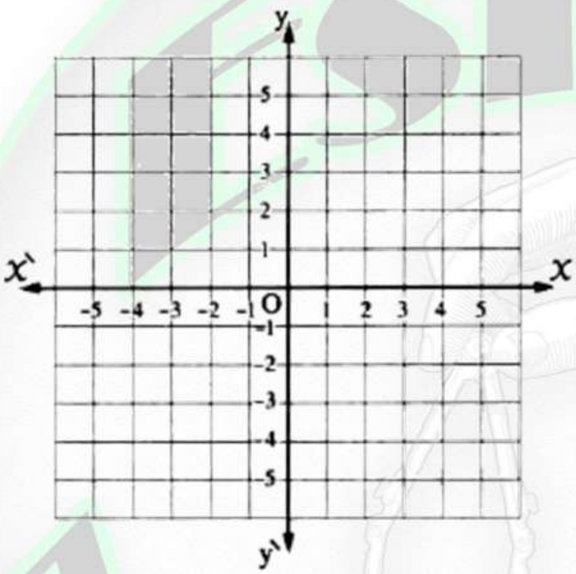

13) $y - 2x = -1$

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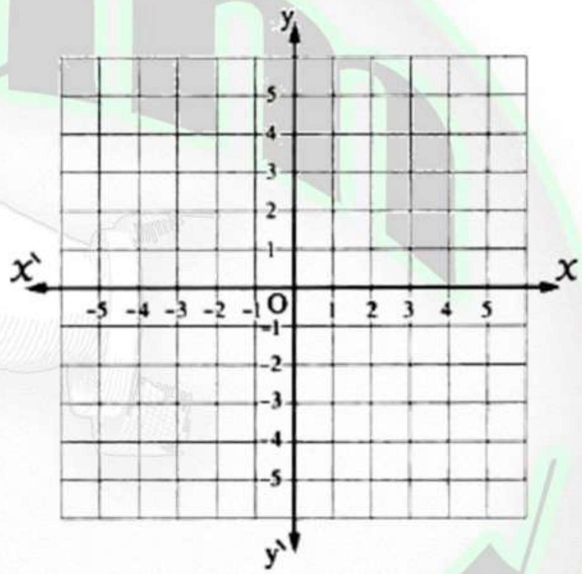
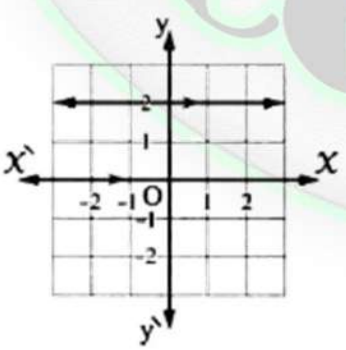
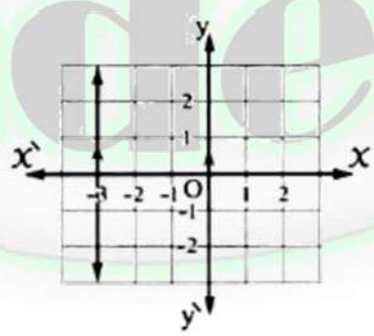


Special cases : a linear relation $ax + by = c$

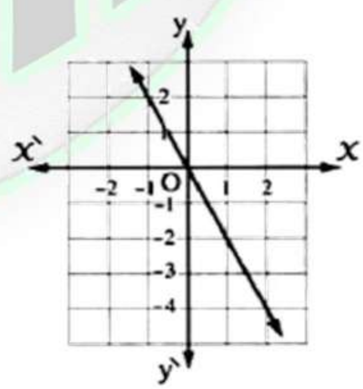
- **If $a = 0$, $b \neq 0$**



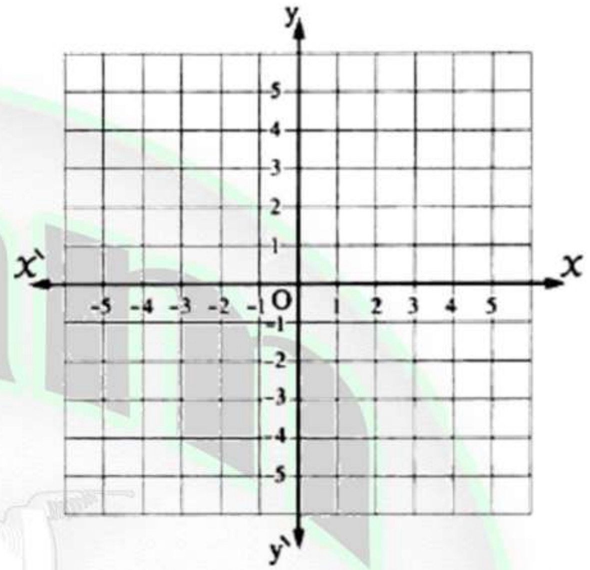
- **If $b = 0$, $a \neq 0$**



- **If $c = 0$**



14) Graph the straight line which represents the relation : $2x + 5y = 10$ and if this straight line intersects x-axis at the point A and y-axis at the point B , find the area of Δ OAB where O is the origin point.



2-2 Slope of straight line

- The slope of the straight line $= \dfrac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \dfrac{\text{the vertical change}}{\text{the horizontal change}}$
- $S = \dfrac{y_2 - y_1}{x_2 - x_1}$

Find the slope of the straight line passing through each pair of points in the following :

1) A = (1 , 2) , B = (3 , 3)

2) (2 , 4) , (4 , 5)

3) (– 2 , – 3) , (– 4 , 1)

4) (1 , 3) , (4 , 2)

5) (3 , 1) , (– 1 , 0)

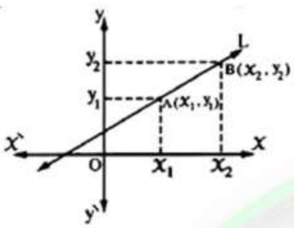
6) (2 , 1) , (3 , 4)

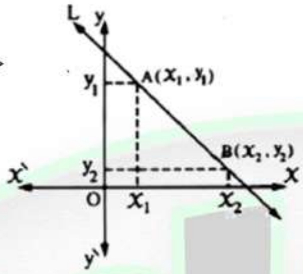
7) (3 , – 5) , (– 4 , 2)

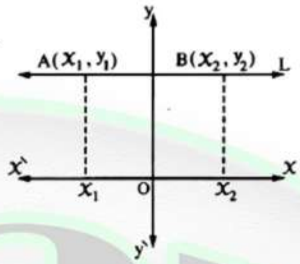
8) (– 3 , – 1) , (1 , 0)

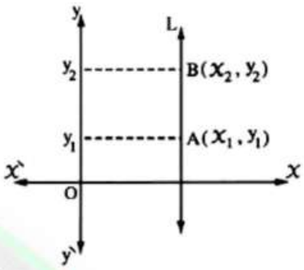
9) (– 6 , 3) , (– 4 , 2)

Remarks

➢ 

➢ 

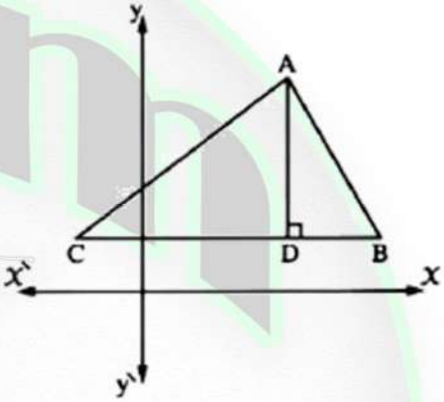
➢ 

➢ 

10) **In the opposite figure :**

ABC is a triangle in which $\overline{BC} \parallel \overleftrightarrow{XX`}$, $\overline{AD} \perp \overline{BC}$

Complete the following by choosing one of the words (positive , negative , zero , undefined) in the spaces :



a. The slope of $\overleftrightarrow{AB}$ is ....

b. The slope of $\overleftrightarrow{AC}$ is ....

c. The slope of $\overleftrightarrow{BC}$ is ....

d. The slope of $\overleftrightarrow{AD}$ is ....

11) **If the slope of the straight line passing through the two points (– 3 , 4) and (1 , y) is 2 , find the value of y**

....

....

....

....

12) If the slope of the straight line passing through the two points (3 , – 1) , (7 , a) is $\frac{3}{4}$, **find the value of a :**

13) **Prove that :** The points : A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.

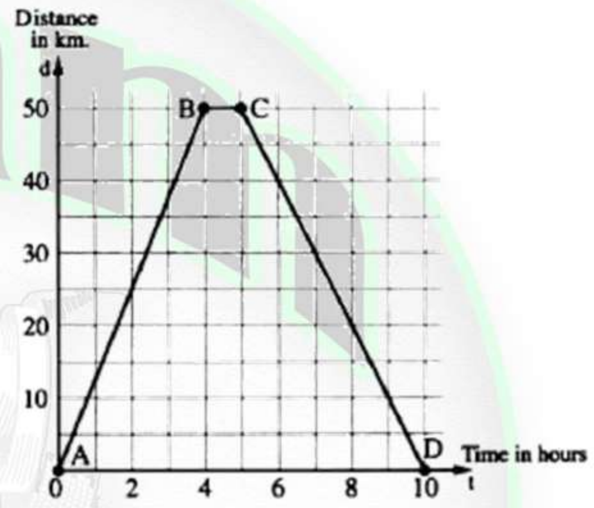
14) **Prove that :** C (– 1 , 2) $\in \overleftrightarrow{AB}$, where A (1 , 3) and B (3 , 4)

15) **If the points : A , B and C are collinear where : A (3 , 2) , B (5 , – 1) and C (1 , k) , find the value of k**

2-3 Real life applications on the slope

➢ The slope of the straight line which represents this relation = $\frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}}$

1) **Waleed rode his bicycle from Cairo to Benha , then he returned back to Cairo. The opposite graph represents the bicycle motion during going and returning back :**



a. Find his velocity in going trip.

....

....

b. Find his velocity in returning back trip.

....

....

c. Find the average velocity during all trips.

....

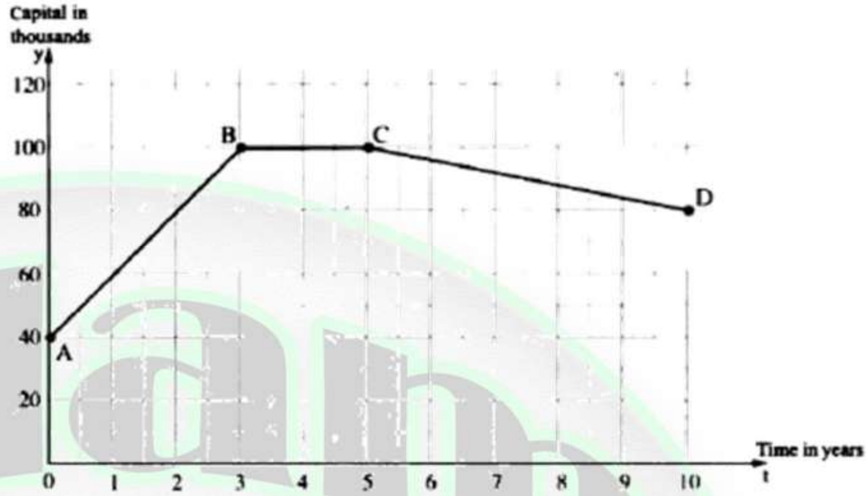
....

d. What do you say about the horizontal line segment in the graph ?

....

....

2) **The opposite graph shows the change of the capital of a company within 10 years :**



a. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$ What is the meaning of each of them ?

....

....

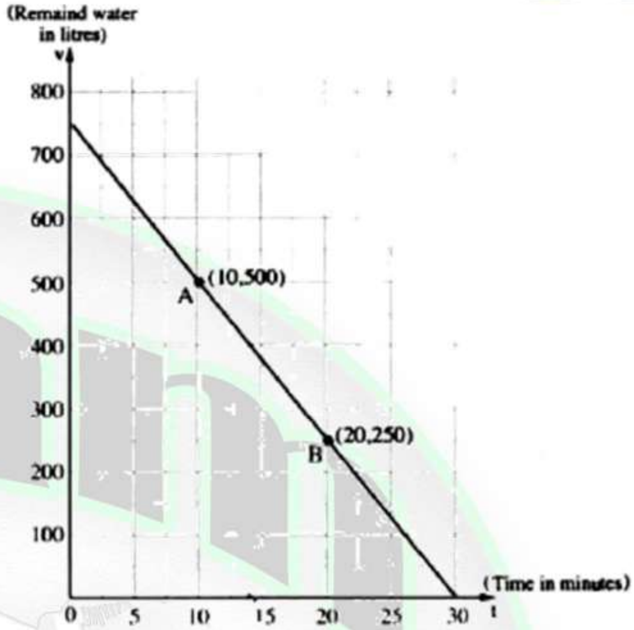
....

....

b. Calculate the capital of the company at the beginning.

....

3) A tank of water is filled with water completely. A tap is opened below the tank to empty it , the opposite graph represents the relation between the time (t) in minutes and the amount of water remained in the tank (v) in litres :



a. What is the greatest capacity of the tank ?

....

b. What is the time needed to empty the tank?

....

c. What is the amount remained in the tank after 20 minutes ?

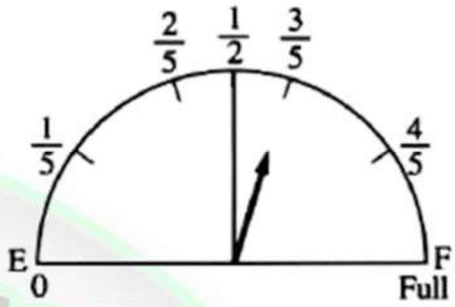
....

d. What is the rate of emptying the tank ?

....

....

4) Hossam filled the tank of his car with fuel given that its capacity is 50 litres.
After Hossam covered a distance = 200 km. ,
he noticed that fuel meter shows that the tank has fuel = $\frac{3}{5}$ its capacity.
Graph the relation between the distance covered by the car and the amount of fuel in the tank and calculate the distance covered by the car till the tank becomes empty.



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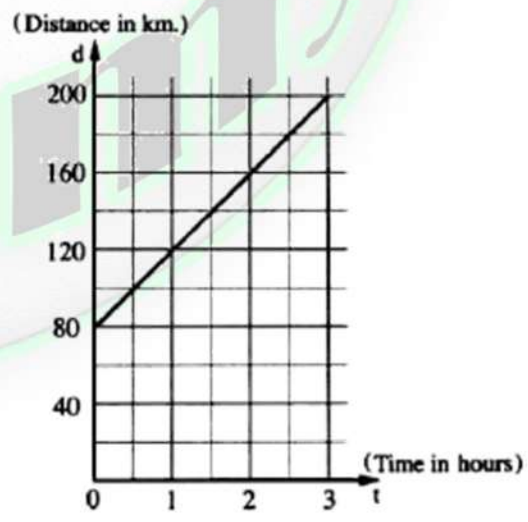
5) **The opposite graph represents the motion of a car measured from a fixed point A :**



a. Determine the uniform velocity of the car.

....

b. Calculate the covered distance after two hours

....

((Sheet 5))

" Operations on the real numbers "

Find each of the following in simplest form :

1) $\sqrt{2} + 3\sqrt{2} + 2\sqrt{2} =$ ....

2) $5\sqrt{3} - 2\sqrt{3} + 4\sqrt{3} =$ ....

3) $\sqrt{5} - \sqrt{3} + 2\sqrt{5} + \sqrt{3} =$ ....

4) $3\sqrt{2} - 2\sqrt{5} + 5\sqrt{2} + \sqrt{5} =$ ....

5) $\sqrt{3} \times \sqrt{3} =$ ....

6) $\sqrt{2} \times \sqrt{3} =$ ....

7) $2\sqrt{2} \times 3\sqrt{5} =$ ....

8) $2\sqrt{2} \times 3\sqrt{2} =$ ....

9) $\sqrt{2}\ (5 + \sqrt{2}) =$ ....

10) $(\sqrt{2} + 1) (\sqrt{3} + 2) =$ ....

Put the denominator as whole number :

1) $\frac{10}{\sqrt{5}}$ 2) $\frac{2}{3\sqrt{2}}$ 3) $\frac{\sqrt{2}+3}{\sqrt{2}}$

Complete :

1) The additive inverse of $\frac{6}{\sqrt{2}} =$ ....

2) The additive inverse of $(\sqrt{2} - \sqrt{5}) =$ ....

3) The multiplicative inverse of $\sqrt{5}$ is ....

4) The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is ....

((Sheet 6))

" Operations on the square roots "

Find in simplest form :

1) $\sqrt{2} + \sqrt{18} + \sqrt{8}$

2) $\sqrt{98} - \sqrt{128} - \sqrt{18} + 4\sqrt{2}$

3) $2\sqrt{3} + \sqrt{27} - \sqrt{48}$

4) $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

5) $2\sqrt{5} + 4\sqrt{20} - 5\sqrt{\frac{1}{5}}$

6) $\sqrt{3} + \frac{3}{\sqrt{3}} - \sqrt{2} \times \sqrt{6}$

7) $\sqrt{27} + \sqrt{8} - 2\sqrt{12} + \sqrt{18}$

8) $\sqrt{3} + 2\sqrt{20} + \sqrt{12} + \sqrt{45}$

Complete :

1) If $X = \frac{\sqrt{6}}{\sqrt{2}}$ Then $X^{-1} =$ ....

2) $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ... in the same pattern

((Sheet 7))

" The two conjugate numbers "

[[Note]]

1) $(X+3), (X-3)$ are conj
2) $(\sqrt{3}+\sqrt{2}), (\sqrt{3}-\sqrt{2})$ are conj
3) $(\sqrt{5}-1), (\sqrt{5}+1)$ are conj
4) $(\sqrt{5}+\sqrt{3}), (\sqrt{5}+\sqrt{3})$ not conj

1. If $X = \frac{2}{\sqrt{7}-\sqrt{5}}$ $Y = \sqrt{7} - \sqrt{5}$, Find $(X+Y)^2$

2. If $X = \sqrt{5} - \sqrt{2}$ $Y = \frac{3}{\sqrt{5}-\sqrt{2}}$ Prove that X and Y are conjugate numbers

   then Find $X^2 - 2XY + Y^2$

3. If $X = \sqrt{7} + \sqrt{5}$ $Y = \frac{2}{X}$

   Find $\frac{X+Y}{XY}$

4. If $X = \frac{4}{\sqrt{7}-\sqrt{3}}$ and $Y^{-1} = \frac{1}{\sqrt{7}-\sqrt{3}}$ Find X^2Y^2

((Sheet 8))

" Operations on the cube roots "

Find in simplest form :

1) $\sqrt[3]{2} + \sqrt[3]{16} + 2\sqrt[3]{54}$

2) $\sqrt[3]{24} - 2\sqrt[3]{3} + \sqrt[3]{81}$

3) $\sqrt[3]{-54} + \sqrt[3]{16} - \sqrt[3]{250}$

4) $\sqrt[3]{81} + \sqrt[3]{-24} - 3\sqrt[3]{\frac{1}{9}}$

5) $\sqrt[3]{108} - 2\sqrt[3]{4} - \sqrt[3]{\frac{1}{2}}$

6) $\sqrt[3]{3} - \sqrt[3]{4} \times \sqrt[3]{6} + 3\sqrt[3]{\frac{1}{9}}$

7) $\frac{7}{3}\sqrt{18} + \sqrt[3]{54} - 7\sqrt{2} + \sqrt[3]{16}$

8) $\sqrt[3]{-16} + \frac{14}{\sqrt{7}} - \sqrt{28} + \sqrt[3]{54}$

((Sheet 9))

" Applications on the real numbers "

Important rules :

[[Cube]]

$L.S.A = 4\,L^2$

$T.S.A = 6\,L^2$

$Volume = L^3$

[[Cuboid]]

$L.S.A = 2\,(X + Y) \times Z$

$T.S.A = 2\,(XY + YZ + ZX)$

$Volume = XYZ$

[[Circle]]

$Circumference = 2\,\Pi\,r$

$Area = \Pi\,r^2$

[[Sphere]]

$Volume = \frac{4}{3}\,\Pi\,r^3$

$Area = 4\,\Pi\,r^2$

[[Right circular cylinder]]

$L.S.A = 2\,\Pi\,rh$

$T.S.A = 2\,\Pi\,rh + 2\,\Pi\,r^2$

$Volume = \Pi\,r^2\,h$

Complete :

1) If the edge of a cube is 5 cm then its volume = ……………….cm^3 .
2) If the volume of cube 64 cm^3 . Then its lateral area = ……………cm^2
3) If the total area of cube 96 cm^2 . Then the area of one face = ………..cm^2
4) Aright circular cylinder with volume 40π cm^3 and its height 10 cm then its base radius = ……………….
5) The volume sphere whose diameter 6 cm = ………..cm^3 .
6) If the volume of sphere $\frac{9}{16}\pi$ cm^3 . Then its radius = …………… cm .

Problems :

1) A cube whose lateral area is 36 cm^2 . Find its total area and its volume .
2) A cube its volume 27 cm^3 . Find its total area .
3) The sum of all edges of a cube is 60 cm . Find its volume .
4) Acuboid its dimensions 3 cm , 4 cm , 5 cm . Find its total area and its volume .
5) A circle its area 154 cm^2 . Find its circumference .
6) Aright circular cylinder its volume 924 cm^3 and its height 6 cm . Find the lateral area .
7) Find the height of right circular cylinder whose height is equal to its base radius and its volume is 72π cm^3 .
8) The volume of sphere is 4188 cm^3 . Find its radius length .
9) Ametalic sphere with diameter 6 cm has got melt and changed into circular cylinder with radius 3 cm . Find its height .

((Sheet 10))

" Solving equations and inequalities of first degree in one variable in R "

Find S.S of equations in R

1) $2x - 3 = 4$

2) $\sqrt{5}\,x - 1 = 4$

Find S.S of inequalities in R and graph the S.S on number line :

1) $2x - 1 \geq 3$

2) $2x + 5 \geq 3$

3) $3 - 2x \geq 7$

4) $5 - 3x \leq 11$

5) $-8 \leq 3x + 1 \leq 4$

6) $13 \geq 2x - 1 \geq 5$

7) $|-3| < 2x - 1 < 5$

8) $5 \leq \frac{-2X+6}{3} < 4$

9) $2 + 2x \leq 3x + 3 < 5 + 2x$

10) $x - 1 < 3x - 1 \leq x + 1$

((Sheet 11))

" Relation between two variables "

1) **Find three ordered pairs satisfy this relation :**

$2X + Y = 5$

2) **Represent graphically**

$X + 2Y = 3$ $\qquad\qquad$ $Y - 3X = 1$

3) **Complete :**

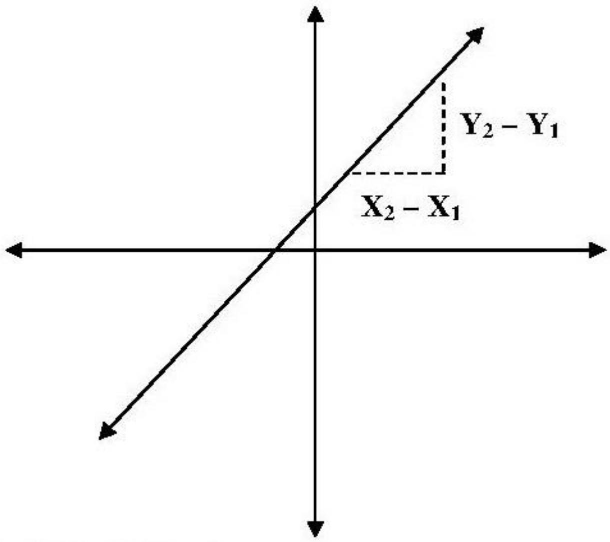
1) If (3 , 6) satisfies $Y = KX$. Then $K =$ ………………….
2) If (3 , 1) satisfies $Y - 3X = a$. Then $a =$ ……………
3) If (3 , a) satisfies $Y - 2X = 4$. Then $a =$ ……………
4) If (K , 2 K) satisfies $X + Y = 15$. Then $K =$ …………
5) If (2 , -5) satisfies $3X - Y + C = 0$. Then $C =$ ………..
6) If the relation $2X + Y = 6$. Then the intersection point of

   X – a x is ……….. and Y – a x is ………………

((Sheet 12))

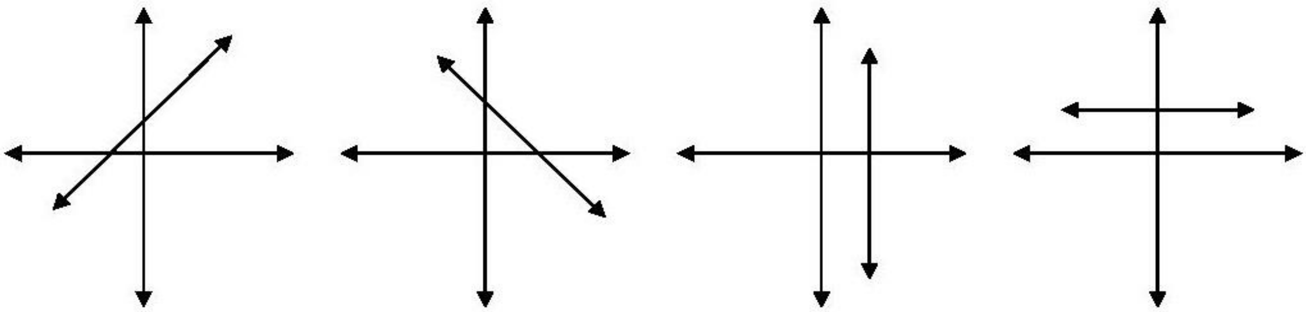
" Slope of straight line "

$$S = \frac{Y_2 - Y_1}{X_2 - X_1}$$



1) <u>Classify the slope of st. line in each of the following</u>

" Positive – negative – zero – undefined "



2) <u>Complete :</u>

1) The slop of any horizontal st. line = …………

2) The slope of any vertical st. line = ……………

3) If A , B , C , are collinear then the slope of $\overleftrightarrow{AB}$ = ……………

4) The slope of st. line which passes through (2 , 3) (5 , 7) is ………………

5) If the st. line which passes through (2 , 3) (5 , k) parallel to X – a x is then K …………

6) If the st. line which passes through (3 , 4) (K , 7) parallel to Y – a x is then K = …………..

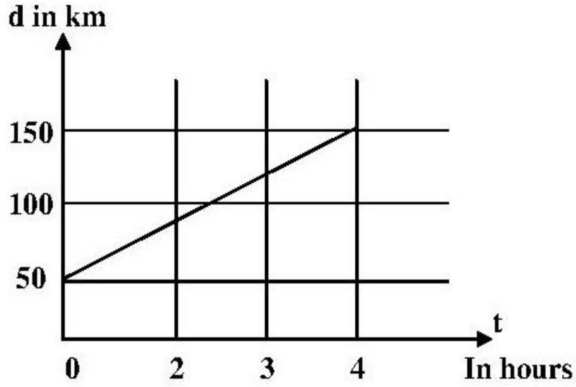
3) If the slope st. line which passes through two points (1 , 3) , (1 , K) equal 3 . Find the value of K .

4) Prove that A , B and C are collinear where A (1 , 1) B (2 , 2) C (-3 , -3)

((Sheet 13))

" Real life applications on the slope "

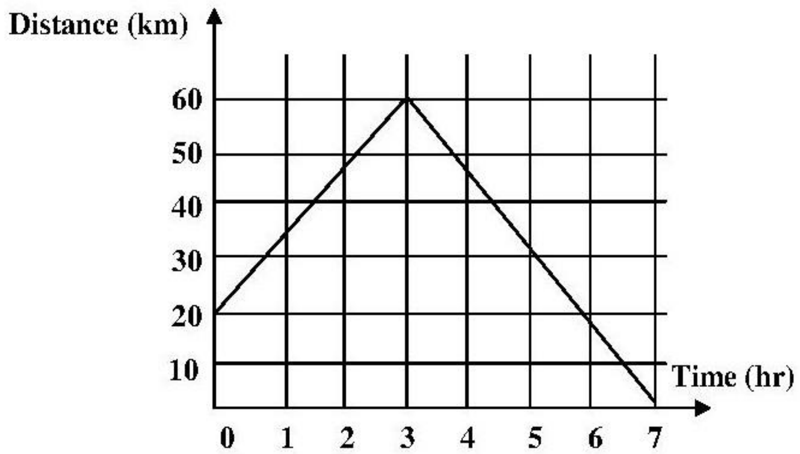
1) The opposite graph represents the motion of a car moving with uniform velocity determine the velocity of the car .



2) The following figure represents the motion of bicycle find the regular speed during

a) The first three hours

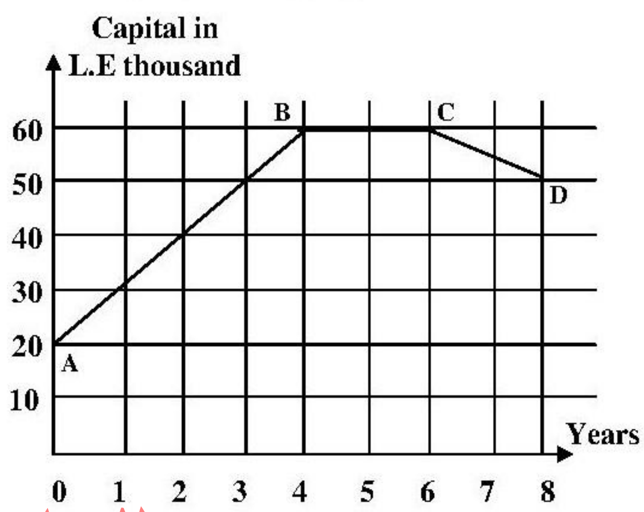
b) The next four hours



3) The opposite figure shows capital change of accompany during 8 years

a) Find the slope of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$

b) Find the starting capital of the company



5) Operations on real numbers

Find each of the following in simplest form :

1) $\sqrt{2} + 3\sqrt{2} + 2\sqrt{2} =$ ……………………

2) $5\sqrt{3} - 2\sqrt{3} + 4\sqrt{3} =$ ……………………

3) $\sqrt{5} - \sqrt{3} + 2\sqrt{5} + \sqrt{3} =$ …………………

4) $3\sqrt{2} - 2\sqrt{5} + 5\sqrt{2} + \sqrt{5} =$ ………………

5) $\sqrt{3} \times \sqrt{3} =$ ……………

6) $\sqrt{2} \times \sqrt{3} =$ ……………

7) $2\sqrt{2} \times 3\sqrt{5} =$ …………

8) $2\sqrt{2} \times 3\sqrt{2} =$ …………

9) $\sqrt{2}\ (5 + \sqrt{2}) =$ ………

10) $(\sqrt{2} + 1)(\sqrt{3} + 2) =$ ……………

Put the denominator as whole number :

1) $\frac{10}{\sqrt{5}}$ 2) $\frac{2}{3\sqrt{2}}$ 3) $\frac{\sqrt{2}+3}{\sqrt{2}}$

Complete :

1) The additive inverse of $\frac{6}{\sqrt{2}} =$ ………………

2) The additive inverse of $(\sqrt{2} - \sqrt{5}) =$ ………………

3) The multiplicative inverse of $\sqrt{5}$ is …………………

4) The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is ………………

6) Operation on square root

Find in simplest form :

1) $\sqrt{2} + \sqrt{18} + \sqrt{8}$

2) $\sqrt{98} - \sqrt{128} - \sqrt{18} + 4\sqrt{2}$

3) $2\sqrt{3} + \sqrt{27} - \sqrt{48}$

4) $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

5) $2\sqrt{5} + 4\sqrt{20} - 5\sqrt{\frac{1}{5}}$

6) $\sqrt{3} + \frac{3}{\sqrt{3}} - \sqrt{2} \times \sqrt{6}$

7) $\sqrt{27} + \sqrt{8} - 2\sqrt{12} + \sqrt{18}$

8) $\sqrt{3} + 2\sqrt{20} + \sqrt{12} + \sqrt{45}$

Complete :

1) If $X = \frac{\sqrt{6}}{\sqrt{2}}$ Then X^{-1} = ………………….

2) $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ………… in the same pattern

7) The conjugate numbers

1. If $X = \frac{2}{\sqrt{7}-\sqrt{5}}$ $Y = \sqrt{7} - \sqrt{5}$, Find $(X + Y)^2$

2. If $X = \sqrt{5} - \sqrt{2}$ $Y = \frac{3}{\sqrt{5}-\sqrt{2}}$ Prove that X and Y are conjugate numbers then Find $X^2 - 2XY + Y^2$

3. If $X = \sqrt{7} + \sqrt{5}$ $Y = \frac{2}{X}$

   Find $\frac{X+Y}{XY}$

4. If $X = \frac{4}{\sqrt{7}-\sqrt{3}}$ and $Y^{-1} = \frac{1}{\sqrt{7}-\sqrt{3}}$ Find X^2Y^2

8) Operation on cube root

Find in simplest form :

1) $\sqrt[3]{2} + \sqrt[3]{16} + 2\sqrt[3]{54}$

2) $\sqrt[3]{24} - 2\sqrt[3]{3} + \sqrt[3]{81}$

3) $\sqrt[3]{-54} + \sqrt[3]{16} - \sqrt[3]{250}$

4) $\sqrt[3]{81} + \sqrt[3]{-24} - 3\sqrt[3]{\frac{1}{9}}$

5) $\sqrt[3]{108} - 2\sqrt[3]{4} - \sqrt[3]{\frac{1}{2}}$

6) $\sqrt[3]{3} - \sqrt[3]{4} \times \sqrt[3]{6} + 3\sqrt[3]{\frac{1}{9}}$

7) $\frac{7}{3}\sqrt{18} + \sqrt[3]{54} - 7\sqrt{2} + \sqrt[3]{16}$

8) $\sqrt[3]{-16} + \frac{14}{\sqrt{7}} - \sqrt{28} + \sqrt[3]{54}$

9) Application on real numbers

Complete :

1) If the edge of a cube is 5 cm then its volume = ……………….cm^3 .
2) If the volume of cube 64 cm^3 . Then its lateral area = …………….cm^2
3) If the total area of cube 96 cm^2 . Then the area of one face = ………..cm^2
4) Aright circular cylinder with volume 40π cm^3 and its height 10 cm then its base radius = ……………….
5) The volume sphere whose diameter 6 cm = …………cm^3 .
6) If the volume of sphere $\frac{9}{16}\pi$ cm^3 . Then its radius = …………… cm .

Problems :

1) A cube whose lateral area is 36 cm^2 . Find its total area and its volume .
2) A cube its volume 27 cm^3 . Find its total area .
3) The sum of all edges of a cube is 60 cm . Find its volume .
4) Acuboid its dimensions 3 cm , 4 cm , 5 cm . Find its total area and its volume .
5) A circle its area 154 cm^2 . Find its circumference .
6) Aright circular cylinder its volume 924 cm^3 and its height 6 cm . Find the lateral area .
7) Find the height of right circular cylinder whose height is equal to its base radius and its volume is 72π cm^3 .

8) The volume of sphere is 4188 cm^3 . Find its radius length .

9) Ametalic sphere with diameter 6 cm has got melt and changed into circular cylinder with radius 3 cm . Find its height .

10) Solving equations and inequality

1) $2X - 1 \geq 3$

2) $2X + 5 \geq 3$

3) $3 - 2X \geq 7$

4) $5 - 3X \leq 11$

7) $|-3| < 2X - 1 < 5$

8) $5 \leq \frac{-2X+6}{3} < 4$

9) $2 + 2X \leq 3X + 3 < 5 + 2X$

10) $X - 1 < 3X - 1 \leq X + 1$

Unit 2

1) Relation between two variables

1) **Find three ordered pairs satisfy this relation :**

$2X + Y = 5$

2) **Represent graphically**

$X + 2Y = 3$ $Y - 3X = 1$

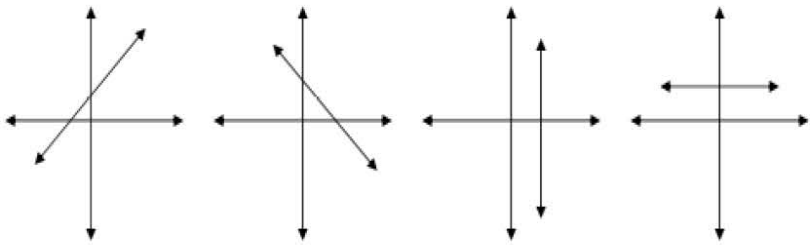
3) **Complete :**

1) If (3 , 6) satisfies $Y = KX$. Then K = ………………….
2) If (3 , 1) satisfies $Y - 3X = a$. Then a = …………….
3) If (3 , a) satisfies $Y - 2X = 4$. Then a = …………….
4) If (K , 2 K) satisfies $X + Y = 15$. Then K = ………….
5) If (2 , -5) satisfies $3X - Y + C = 0$. Then C = ………..
6) If the relation $2X + Y = 6$. Then the intersection point of X – a x is ……….. and Y – a x is ………………

2) Slope of straight line

1) Classify the slope of st. line in each of the following

" Positive – negative – zero – undefined "



2) Complete :

1) The slop of any horizontal st. line = ………….

2) The slope of any vertical st. line = ……………

3) If A , B , C , are collinear then the slope of $\overleftrightarrow{AB}$ = ……………

4) The slope of st. line which passes through (2 , 3) (5 , 7) is ………………

5) If the st. line which passes through (2 , 3) (5 , k) parallel to X – a x is then K …………

6) If the st. line which passes through (3 , 4) (K , 7) parallel to Y – a x is then K = …………..

3) If the slope st. line which passes through two points (1 , 3) , (1 , K) equal 3 . Find the value of K .

4) Prove that A , B and C are collinear where A (1 , 1) B (2 , 2) C (-3 , -3)

Sheet (5)
Corollaries of the isosceles triangle theorems

Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the opposite figure :

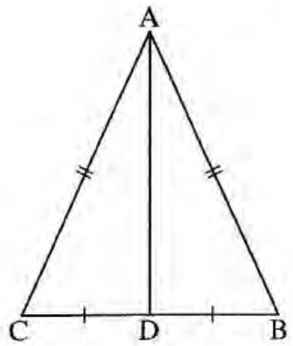
ABC is a triangle in which AB = AC and

$\overline{AD}$ is a median , then :

1 $\overrightarrow{AD}$ bisects $\angle$ BAC

i.e. m ($\angle$ BAD) = m ($\angle$ CAD)

2 $\overline{AD} \perp \overline{BC}$



Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the opposite figure :

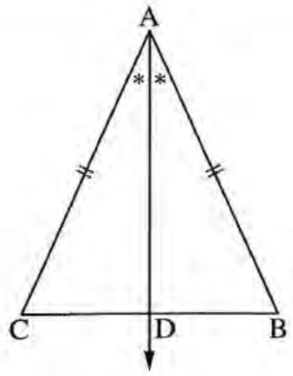
ABC is a triangle in which AB = AC and

$\overrightarrow{AD}$ bisects $\angle$ BAC , then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 $\overline{AD} \perp \overline{BC}$



Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the opposite figure :

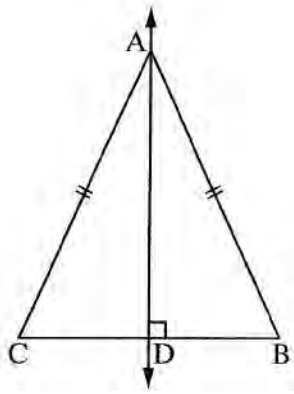
ABC is a triangle in which AB = AC and

$\overleftrightarrow{AD} \perp \overline{BC}$, then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 m ($\angle$ BAD) = m ($\angle$ CAD)



Notice that :

The previous three corollaries can be proved using the congruence of Δ ABD and Δ ACD

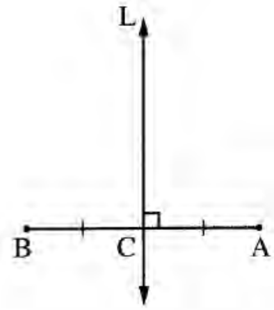
Axis of symmetry of a line segment

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

In the opposite figure :

If the straight line L $\perp$ $\overline{AB}$ and C $\in$ the straight line L where C is the midpoint of $\overline{AB}$, then the straight line L is called the axis of $\overline{AB}$



Property

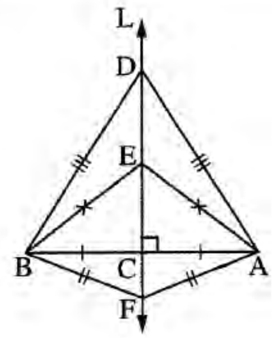
Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

In the opposite figure :

If the straight line L is the axis of $\overline{AB}$,

D $\in$ L , E $\in$ L and F $\in$ L , then

DA = DB , EA = EB and FA = FB

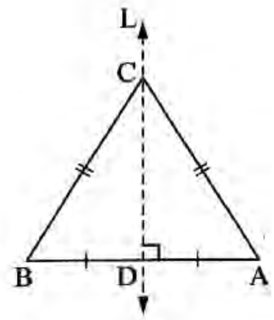


The converse of the previous property is true

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

In the opposite figure :

If C is a point such that CA = CB , then the point C lies on the axis of $\overline{AB}$



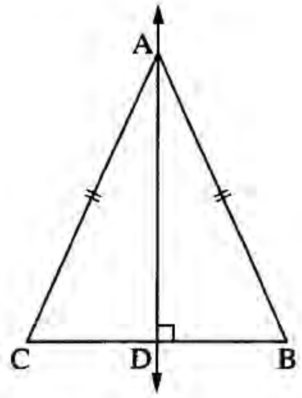
Axis of symmetry of the isosceles triangle

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

For example:

If ABC is an isosceles triangle where $AB = AC$ and $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is called the axis of symmetry of the isosceles triangle ABC

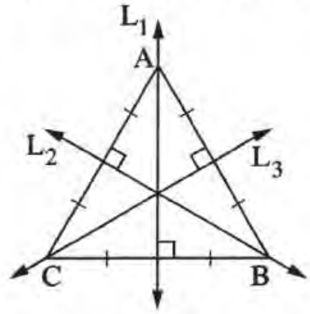


Remarks

1 The equilateral triangle has three axes of symmetry , they are the three perpendiculars drawn from its vertices to the opposite sides.

In the opposite figure :

The straight lines L_1 , L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC



2 The scalene triangle has no axes of symmetry.

[1] Complete:

1 The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base is ....

2 The median of an isosceles triangle drawn from the vertex bisects .... and is perpendicular to ....

3 The bisector of the vertex angle of an isosceles triangle .... and ....

4 In ΔXYZ : If $XY = XZ$, $\overrightarrow{XL} \perp \overline{YZ}$, then $\overleftrightarrow{XL}$ bisects each of .... and ....

5 The straight line perpendicular to the midpoint of a line segment is called ....

6 In the isosceles triangle if the measure of any angle is 60° , then the number of axis of symmetry ....

7 The number of axes of symmetry of the isosceles triangle equal ....

8 The number of symmetrical line in an scalene triangle = ...

9 The number of the axes of symmetry in an equilateral triangle = ...

10 The number of axes of symmetry of the triangle in which the measures of two angles are $50°$, $70°$ = ...

11 In ΔABC : If $AB = AC$, then the point A lies on the axis of symmetry of ....

12 If D is the midpoint of $\overline{AB}$ and $\overleftrightarrow{CD} \perp \overline{AB}$, then $CA =$ ....

13 The axis of symmetry of the line segment is the straight line which ....

14 Any point on the axis symmetry of a line segment is at two equal distance from ....

15 If the point $A \in$ the axis of symmetry of $\overline{BC}$, then $AB =$ ....

16 The axis of symmetry of isosceles triangle is ....

[2] Essay problems:

1 **In the opposite figure :**

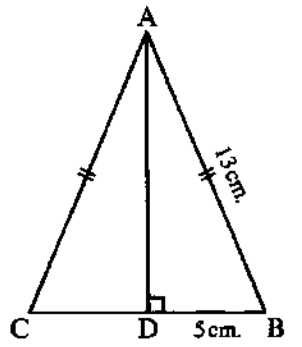
In ΔABC , $AB = AC$,

$\overline{AD} \perp \overline{BC}$,

$AB = 13$ cm. and $BD = 5$ cm.

Find : ❶ The length of $\overline{BC}$

❷ The area of ΔABC



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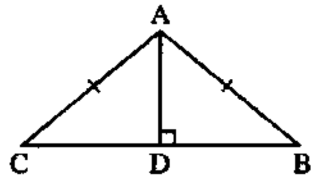
2 **In the opposite figure :**

ABC is a triangle in which : AB = AC , $\overline{AD} \perp \overline{BC}$

m (∠ BAC) = 100° and BD = 3 cm.

Find : (1) m (∠ BAD) (2) The length of $\overline{CB}$



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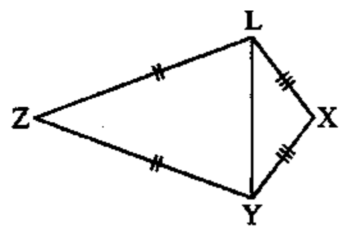
3 **In the opposite figure :**

XL = XY , ZL = ZY ,

M is the midpoint of $\overline{LY}$

Prove that :

X , M , Z are on the same straight line.



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4 **In the opposite figure :**

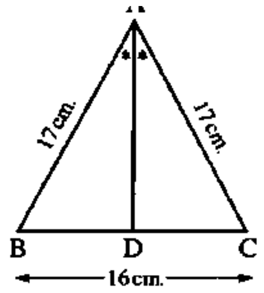
$\overrightarrow{AD}$ bisects $\angle BAC$,

$AB = AC = 17$ cm. ,

and $BC = 16$ cm.

Prove that : $m(\angle ADB) = 90°$,

then find the length of : $\overline{AD}$ and the area of ΔABC



[1] Choose the correct answer:

1 The axis of symmetry of a line segment is the straight line which is ………

(a) Perpendicular to it. (b) its bisector.

(c) parallel to it. (d) the perpendicular bisector.

2 If $A \in$ the axis of symmetry of $\overline{BC}$, then $\overline{AB}$ ……… $\overline{AC}$

(a) $\perp$ (b) $\equiv$ (c) // (d) =

3 If A lies on the axis of symmetry of $\overline{XY}$ then AX …………… AY

(a) // (b) $\perp$ (c) = (d) $\neq$

4 The number of axis of symmetry in the scalene triangle is ………

(a) 1 (b) zero (c) 3 (d) 4

5 The number of axes of symmetry in the isosceles triangle is ………

(a) 1 (b) 2 (c) 3 (d) zero

6 The isosceles triangle has .... axis (axes) of symmetry.

(a) no (b) two (c) only one (d) three

7 The number of axes of symmetry in the equilateral triangle is ....

(a) 0 (b) 2 (c) 3 (d) 1

8 The equilateral triangle has .... axes of symmetry.

(a) one (b) two (c) three (d) otherwise

9 The triangle which has no axes of symmetry is .... triangles.

(a) scalene (b) isosceles (c) equilateral (d) otherwise

10 If Δ ABC has one axes of symmetry and m ($\angle$ ABC) = 140° , then m ($\angle$ A) = ...

(a) 30° (b) 20° (c) 40° (d) 60°

11 The triangle which has three axes of symmetry is .... triangle.

(a) scalene (b) isosceles (c) right-angled (d) equilateral

12 Δ ABC in which m ($\angle$ A) = m ($\angle$ B) = 65° , then it has .... axis (axes) of symmetry.

(a) 1 (b) 2 (c) 3 (d) zero

13 In Δ ABC if : m ($\angle$ A) = 40° and m ($\angle$ B) = 70° , then Δ ABC has .... axis (axes) of symmetry.

(a) 3 (b) 1 (c) 2 (d) zero

14 The quadrilateral ABCD in which $\overleftrightarrow{BD}$ is an axis of symmetry of $\overline{AC}$ may by ....

(a) a rhombus (b) a rectangle (c) a parallelogram (d) a trapezium

15 Δ ABC , AB = AC , D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is ....

(a) median. (b) altitude.

(c) bisector of the vertex angle. (d) all the previous.

[2] Essay problems:

1 **In the opposite figure :**

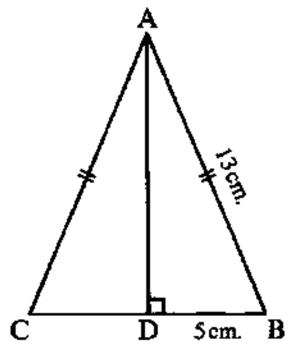
In ΔABC , $AB = AC$,

$\overline{AD} \perp \overline{BC}$,

$AB = 13$ cm. and $BD = 5$ cm.

Find : ❶ The length of $\overline{BC}$

❷ The area of ΔABC

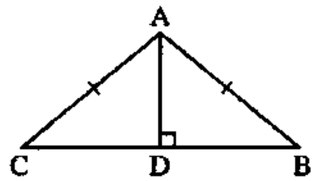


2 **In the opposite figure :**

ABC is a triangle in which : $AB = AC$, $\overline{AD} \perp \overline{BC}$

$m(\angle BAC) = 100°$ and $BD = 3$ cm.

Find : (1) $m(\angle BAD)$ (2) The length of $\overline{CB}$



3 **In the opposite figure :**

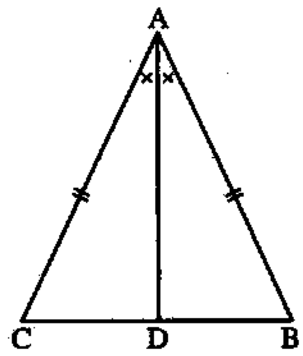
In ΔABC :

$AB = AC$, $\overrightarrow{AD}$ bisects $\angle BAC$

and $BD = 3$ cm.

Prove that : $\overline{AD} \perp \overline{BC}$

, then find the length of : $\overline{CB}$



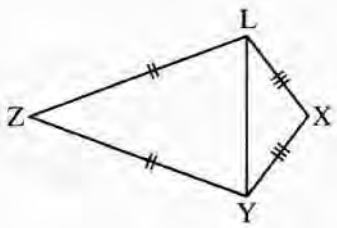
4 **In the opposite figure :**

$XL = XY$, $ZL = ZY$,

M is the midpoint of $\overline{LY}$

Prove that :

X , M , Z are on the same straight line.



4-6 Corollaries of isosceles triangle theorems

- **Corollary (1)**

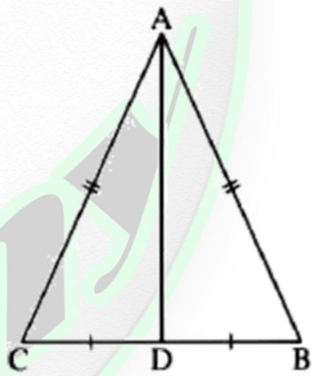
  The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

- **Corollary (2)**

  The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

- **Corollary (3)**

  The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



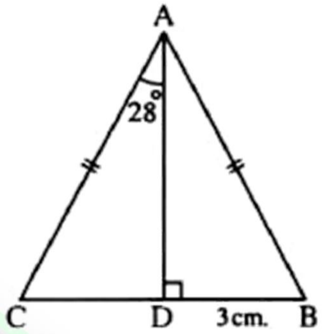
1) **In the opposite figure :**

ABC is an isosceles triangle where

AB = AC and D $\in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$,

m ($\angle$ CAD) = 28° and BD = 3 cm. Find :

1 m ($\angle$ BAC) **2** the length of $\overline{BC}$



➢ **Axis of symmetry of a line segment**

Definition

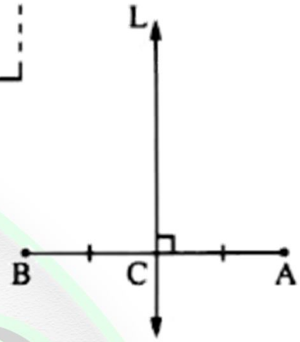
The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

➢ **Property**

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

➢ **The converse of the previous property is true**

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.



2) **In the opposite figure :**

The straight line L is the axis of $\overline{AB}$

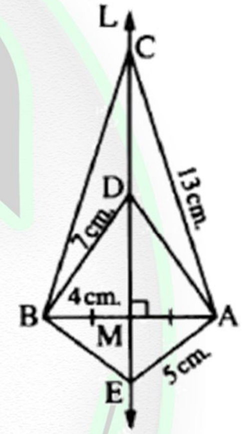
If the points C , D and E belong to the straight line L , $L \cap \overline{AB} = \{M\}$

where AC = 13 cm. ,

DB = 7 cm. , AE = 5 cm. and

MB = 4 cm. Find the length of each of

$\overline{CB}$, $\overline{DA}$, $\overline{EB}$ and $\overline{MA}$



3) ΔABC is an isosceles triangle where $AB = AC$, $\overrightarrow{BX}$ bisects $\angle ABC$ and intersects $\overline{AC}$ at X , $\overrightarrow{CY}$ bisects $\angle ACB$ and intersects $\overline{AB}$ at Y If $\overrightarrow{BX} \cap \overrightarrow{CY} = \{M\}$,

prove that : $\overleftrightarrow{AM} \perp \overline{BC}$

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Axis of symmetry of the isosceles triangle

The isosceles triangle has one axis of symmetry.
It is the straight line drawn from the vertex angle perpendicular to its base.

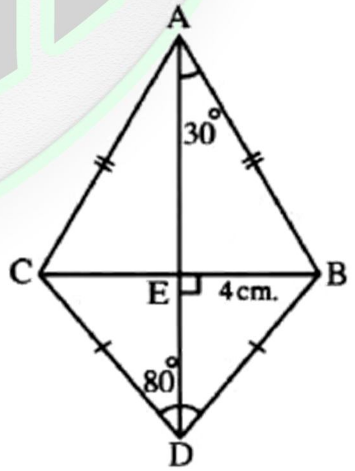
4) **In the opposite figure :**

ABDC is a quadrilateral in which :

$AB = AC$, $BD = CD$, $\overline{AD} \perp \overline{BC}$,

$\overline{AD} \cap \overline{BC} = \{E\}$, $m(\angle BAD) = 30°$,

$m(\angle BDC) = 80°$ and $BE = 4$ cm.



Complete the following :

1 $m(\angle DAC) =$ .... °

2 $m(\angle BDE) =$ .... °

3 $m(\angle ACB) =$ ....°

4 $EC =$ .... cm.

5 $AC =$ .... cm.

5-1 Inequality

➢ **Axioms of inequality relation**

For any four numbers a , b , c and d

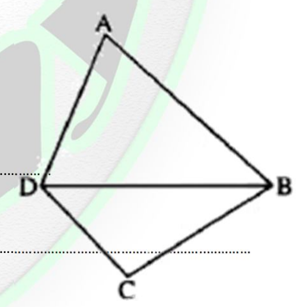
1. If $a > b$, then $a + c > b + c$
2. If $a > b$, then $a - c > b - c$
3. If $a > b$, $c > 0$, then $a c > b c$
4. If $a > b$, $b > c$, then $a > c$
5. If $a > b$, $c > d$, then $a + c > b + d$

1) **In the opposite figure :**

If B and C belong to $\overline{AD}$ such that $AB > CD$ **Prove that :** $AC > BD$

2) **In the opposite figure :** If $m(\angle ADB) > m(\angle ABD)$,

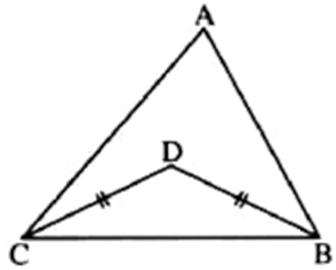
$m(\angle CBD) < m(\angle CDB)$ **Prove that :** $m(\angle ADC) > m(\angle ABC)$



3) **In the opposite figure :**

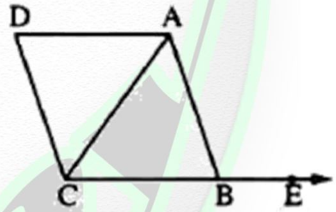
If m (∠ ABC) > m (∠ ACB) and BD = DC

Prove that : m (∠ ABD) > m (∠ ACD)



4) **In the opposite figure :** ABCD is a parallelogram

$E \in \overrightarrow{CB}$ **prove that :** m (∠ ABE) > m (∠ ACD)



LESSON (5) conversemof the isosceles triangle theorems

Mechanism : Isosceles Triangle : Median

Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the opposite figure :

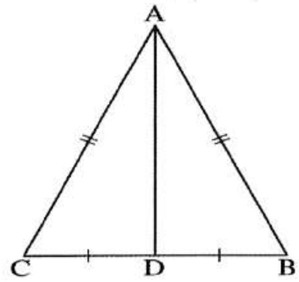
ABC is a triangle in which AB = AC and

$\overline{AD}$ is a median , then :

1 $\overrightarrow{AD}$ bisects $\angle$ BAC

i.e. m ($\angle$ BAD) = m ($\angle$ CAD)

2 $\overline{AD} \perp \overline{BC}$



Mechanism : Isosceles Triangle : Vertex Bisector

Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

n the opposite figure :

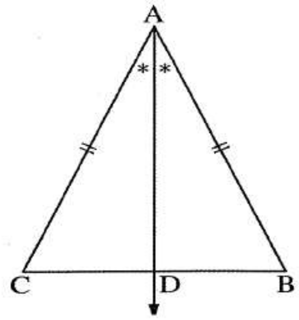
ABC is a triangle in which AB = AC and

$\overrightarrow{AD}$ bisects $\angle$ BAC , then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 $\overline{AD} \perp \overline{BC}$



Mechanism : Isosceles Triangle : Perpendicular

Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the opposite figure :

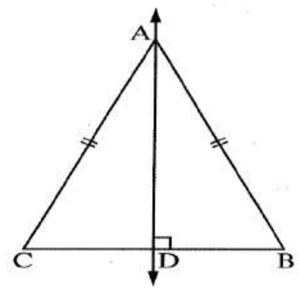
ABC is a triangle in which AB = AC and

$\overleftrightarrow{AD} \perp \overline{BC}$, then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 m ($\angle$ BAD) = m ($\angle$ CAD)



Notice that :

The previous three corollaries can be proved using the congruence of Δ ABD and Δ ACD

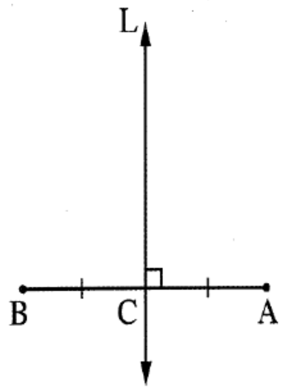
Mechanism: Axis of symmetry of line segment (1)

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

In the opposite figure :

If the straight line $L \perp \overline{AB}$ and $C \in$ the straight line L where C is the midpoint of $\overline{AB}$, then the straight line L is called the axis of $\overline{AB}$



Mechanism: Axis of symmetry of line segment (2)

Property

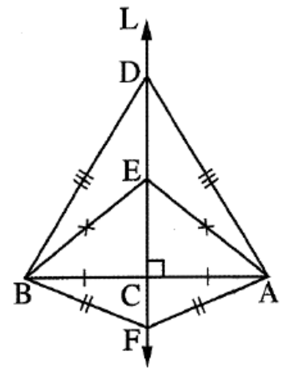
Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

In the opposite figure :

If the straight line L is the axis of $\overline{AB}$,

$D \in L$, $E \in L$ and $F \in L$, then

$DA = DB$, $EA = EB$ and $FA = FB$

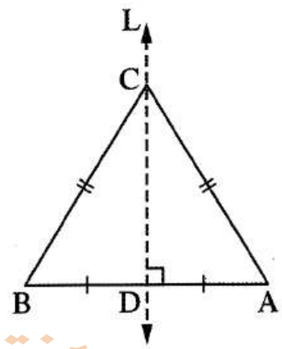


The converse of the previous property is true

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

In the opposite figure :

If C is a point such that $CA = CB$, then the point C lies on the axis of $\overline{AB}$



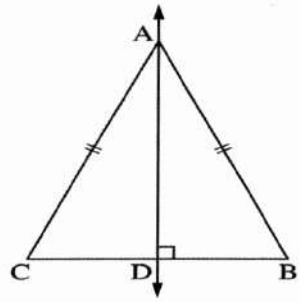
Mechanism: Axis of symmetry of Isosceles Triangle

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

For example:

If ABC is an isosceles triangle where $AB = AC$ and $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is called the axis of symmetry of the isosceles triangle ABC



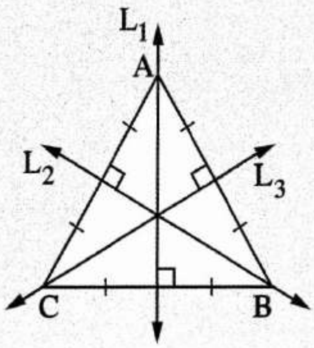
Mechanism : Axis of Symmetry of Equilateral Triangle

Remarks

1 The equilateral triangle has three axes of symmetry, they are the three perpendiculars drawn from its vertices to the opposite sides.

In the opposite figure :

The straight lines L_1, L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC



2 The scalene triangle has no axes of symmetry.

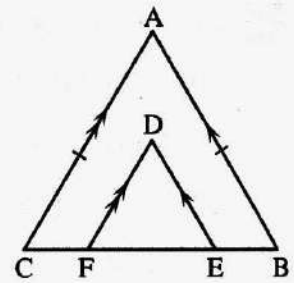
Examples on Part (1) : Isosceles Triangle

Find the perimeter of the figure : ADME

⑦ **In the opposite figure :**

$AB = AC$, $\overline{DE} \parallel \overline{AB}$, $\overline{DF} \parallel \overline{AC}$

Prove that : $DE = DF$



Solution

In Δ ABC

$\therefore$ m $\angle$B = m $\angle$C

$\because$ AC // DF , CF is a transversal

$\because$ AB // DE , EB is a transversal

$\therefore$ m $\angle$B = m $\angle$DEF (Corresponding)

$\because$ AB = AC

$\therefore$ m $\angle$C = m $\angle$DFE (Corresponding)

$\because$ In Δ DEF

$\therefore$ m $\angle$DEF = m $\angle$DFE

$\therefore$ DE = DF

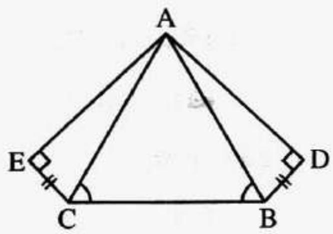
(8) In the opposite figure :

BD = CE

, m (∠ ABC) = m (∠ ACB)

, m (∠ D) = m (∠ E) = 90°

Prove that : m (∠ DAB) = m (∠ CAE)



Solution

In Δ ABC

∵ m ∠ABC = m ∠ACB

∴ AB = AC

In Δ Δ ABD , ACE

1) AB = AC

2) BD = CE

3) m∠D = m∠E = 90°

∴ Δ ABD ≡ Δ ACE

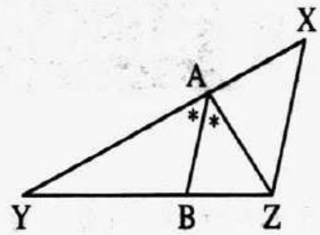
∴ m ∠DAB = m ∠ CAE

(9) In the opposite figure :

$\overrightarrow{AB}$ bisects angle YAZ

, $\overline{AB}$ // $\overline{XZ}$

Prove that : Δ AXZ isosceles triangle.



Solution

In Δ XYZ

∵ AB // XZ , AZ is a transversal

∴ m ∠BAZ = m ∠AZX

(Alternate)

∵ AB // XZ , AX is a transversal

∴ m ∠X = m ∠BAY

(Corresponding)

∵ AB bisects angle YAZ

∴ m ∠YAB = m ∠ZAB

∴ m ∠X = m ∠ AZX

∴ AZ = AX

Δ XYZ is isosceles triangle

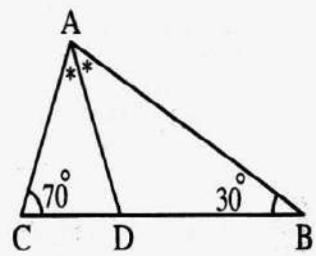
(10) In the opposite figure :

$\overrightarrow{AD}$ bisects ∠ BAC

, m (∠ B) = 30°

, m (∠ C) = 70°

Prove that : Δ ADC is isosceles triangle.



Solution

In Δ ABC

∵ m ∠B = 30°

∵ m ∠C = 70°

∴ m ∠BAC = 180 – 30 – 70 = 80°

∵ AD bisects angle BAC

∴ m ∠BAD = m ∠CAD = 80 ÷ 2 = 40°

∴ In Δ ADC

∴ m ∠ ADC = 180 – 70 – 40 = 70°

∴ m ∠ ADC = m ∠ ACD = 70°

∴ AD = AC

Δ ADC is isosceles triangle

EXERCISES

[A] Complete the Following :

1	The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base is ……………
2	The median of an isosceles triangle drawn from the vertex bisects ………. and is perpendicular to ……….
3	The bisector of the vertex angle of an isosceles triangle ………. and ……….
4	In Δ XYZ : If XY = XZ , $\overrightarrow{XL} \perp \overline{YZ}$, then $\overleftrightarrow{XL}$ bisects each of ………. and ……….
5	The straight line perpendicular to the midpoint of a line segment is called ……….
6	In the isosceles triangle if the measure of any angle is 60° , then the number of axis of symmetry ……….
7	The number of axes of symmetry of the isosceles triangle equal ……….
8	The number of symmetrical line in an scalene triangle = ……….
9	The number of the axes of symmetry in an equilateral triangle = ……….
10	The number of axes of symmetry of the triangle in which the measures of two angles are 50° , 70° = ……….
11	In Δ ABC : If AB = AC , then the point A lies on the axis of symmetry of ……….

12	If D is the midpoint of $\overline{AB}$ and $\overleftrightarrow{CD} \perp \overline{AB}$, then CA = ……………
13	The axis of symmetry of the line segment is the straight line which ……….
14	Any point on the axis symmetry of a line segment is at two equal distance from ……….
15	If the point A ∈ the axis of symmetry of $\overline{BC}$, then AB = ………
16	The axis of symmetry of isosceles triangle is ………

[B] Essay Problems :

1

In the opposite figure :

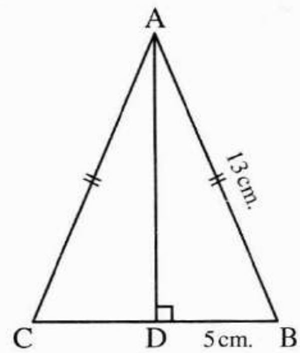
In Δ ABC , AB = AC ,

$\overline{AD} \perp \overline{BC}$,

AB = 13 cm. and BD = 5 cm.

Find : (1) The length of $\overline{BC}$

(2) The area of Δ ABC



2012 Exam (11) Question (5) (b)

2

In the opposite figure :

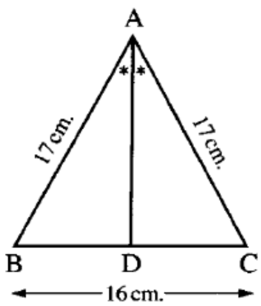
$\overrightarrow{AD}$ bisects ∠ BAC ,

AB = AC = 17 cm. ,

and BC = 16 cm.

Prove that : m (∠ ADB) = 90° ,

then find the length of : $\overline{AD}$ and the area of Δ ABC



2015 Exam (2) Question (4) (a)

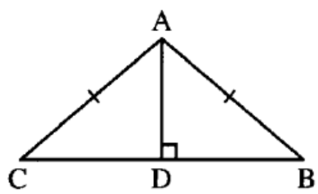
3

In the opposite figure :

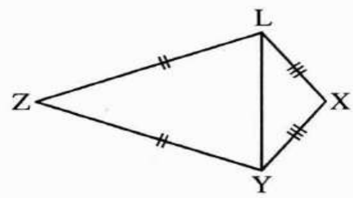
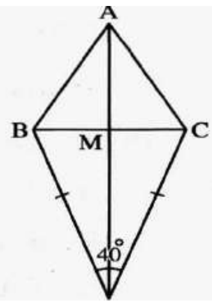
ABC is a triangle in which : AB = AC , $\overline{AD} \perp \overline{BC}$

m (∠ BAC) = 100° and BD = 3 cm.

Find : (1) m (∠ BAD) (2) The length of $\overline{CB}$



2015 Exam (13) Question (4) (a)

4	**In the opposite figure :** XL = XY , ZL = ZY , M is the midpoint of $\overline{LY}$ **Prove that :** X , M , Z are on the same straight line.  **2012 Exam (2) Question (5) (b)**
5	**In the opposite figure :** DB = DC , m (∠ D) = 40° , Δ ABC is an equilateral triangle , if M is the midpoint of $\overline{BC}$ (1) **Find :** m (∠ ABD) (2) **Prove that :** A , M and D are on the same straight line.  **2014 Exam (3) Question (5) (a)**

HOMEWORK

[A] Choose the correct answer:

1	The axis of symmetry of a line segment is the straight line which is ………. (a) Perpendicular to it. (b) its bisector. (c) parallel to it. (d) the perpendicular bisector.
2	If A ∈ the axis of symmetry of $\overline{BC}$, then $\overline{AB}$ ………. $\overline{AC}$ (a) ⊥ (b) ≡ (c) // (d) =
3	If A lies on the axis of symmetry of $\overline{XY}$ then AX …………… AY (a) // (b) ⊥ (c) = (d) ≠
4	The number of axis of symmetry in the scalene triangle is ………. (a) 1 (b) zero (c) 3 (d) 4
5	The number of axes of symmetry in the isosceles triangle is ………. (a) 1 (b) 2 (c) 3 (d) zero

6	The isosceles triangle has .... axis (axes) of symmetry. (a) no (b) two (c) only one (d) three
7	The number of axes of symmetry in the equilateral triangle is .... (a) 0 (b) 2 (c) 3 (d) 1
8	The equilateral triangle has .... axes of symmetry. (a) one (b) two (c) three (d) otherwise
9	The triangle which has no axes of symmetry is .... triangles. (a) scalene (b) isosceles (c) equilateral (d) otherwise
10	If Δ ABC has one axes of symmetry and m ($\angle$ ABC) = 140° , then m ($\angle$ A) = ... (a) 30° (b) 20° (c) 40° (d) 60°
11	The triangle which has three axes of symmetry is .... triangle. (a) scalene (b) isosceles (c) right-angled (d) equilateral
12	Δ ABC in which m ($\angle$ A) = m ($\angle$ B) = 65° , then it has .... axis (axes) of symmetry. (a) 1 (b) 2 (c) 3 (d) zero
13	In Δ ABC if : m ($\angle$ A) = 40° and m ($\angle$ B) = 70° , then Δ ABC has .... axis (axes) of symmetry. (a) 3 (b) 1 (c) 2 (d) zero
14	The quadrilateral ABCD in which $\overleftrightarrow{BD}$ is an axis of symmetry of $\overline{AC}$ may by .... (a) a rhombus (b) a rectangle (c) a parallelogram (d) a trapezium

[B] Essay Problems :

1

In the opposite figure :

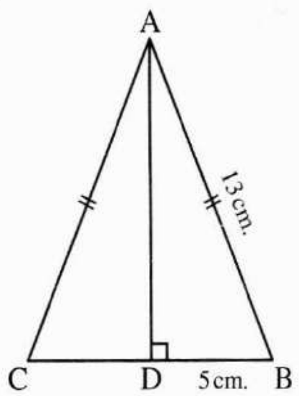
In ΔABC , $AB = AC$,

$\overline{AD} \perp \overline{BC}$,

$AB = 13$ cm. and $BD = 5$ cm.

Find : (1) The length of $\overline{BC}$

(2) The area of ΔABC



2012 Exam (11) Question (5) (b)

2

In the opposite figure :

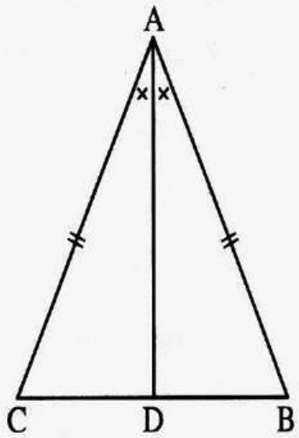
In ΔABC :

$AB = AC$, $\overrightarrow{AD}$ bisects $\angle BAC$

and $BD = 3$ cm.

Prove that : $\overline{AD} \perp \overline{BC}$

, then find the length of : $\overline{CB}$



2014 Exam (1) Question (5) (a)

3

In the opposite figure :

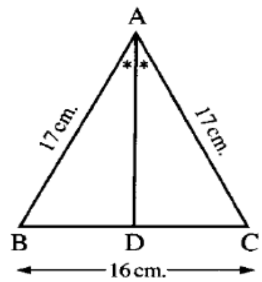
$\overrightarrow{AD}$ bisects $\angle BAC$,

$AB = AC = 17$ cm. ,

and $BC = 16$ cm.

Prove that : $m(\angle ADB) = 90°$,

then find the length of : $\overline{AD}$ and the area of ΔABC



2015 Exam (2) Question (4) (a)

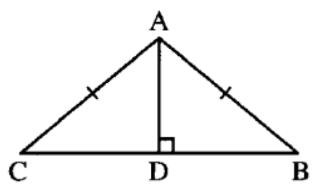
4

In the opposite figure :

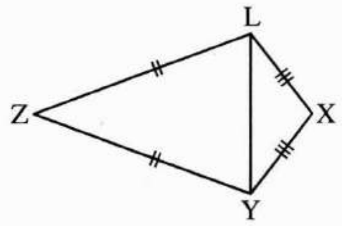
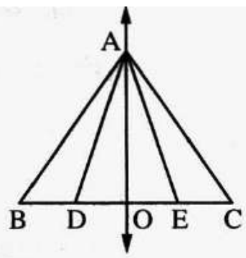
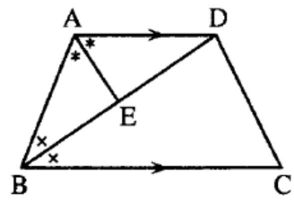
ABC is a triangle in which : $AB = AC$, $\overline{AD} \perp \overline{BC}$

$m(\angle BAC) = 100°$ and $BD = 3$ cm.

Find : (1) $m(\angle BAD)$ (2) The length of $\overline{CB}$



2015 Exam (13) Question (4) (a)

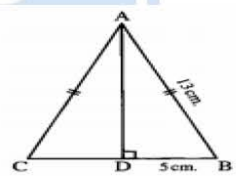
5	**In the opposite figure :** XL = XY , ZL = ZY , M is the midpoint of $\overline{LY}$ **Prove that :** X , M , Z are on the same straight line.  **2012 Exam (2) Question (5) (b)**
6	**In the opposite figure :** The two triangles ABC , AED have the same line of symmetry $\overleftrightarrow{AO}$ What type of each them according to its sides ? **Prove that :** BD = EC  **2014 Exam (10) Question (4) (b)**
7	**In the opposite figure :** ABCD is a quadrilateral in which $\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ bisects ∠ ABC , $\overrightarrow{AE}$ bisects ∠ BAD **Prove that :** (1) AB = AD (2) $\overline{AE} \perp \overline{BD}$ (3) BE = ED  **2015 Exam (11) Question (5) (a)**

Corollaries of isosceles triangle

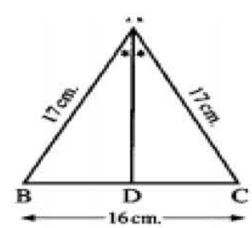
[1] Complete:

1	The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base is ....
2	The median of an isosceles triangle drawn from the vertex bisects .... and is perpendicular to ....
3	The bisector of the vertex angle of an isosceles triangle .... and ....
4	In Δ XYZ : If XY = XZ , $\overrightarrow{XL} \perp \overline{YZ}$, then $\overleftrightarrow{XL}$ bisects each of .... and ....
5	The straight line perpendicular to the midpoint of a line segment is called ....
6	In the isosceles triangle if the measure of any angle is 60° , then the number of axis of symmetry ....
7	The number of axes of symmetry of the isosceles triangle equal ....
8	The number of symmetrical line in an scalene triangle = ...

In the opposite figure :
In Δ ABC , AB = AC ,
$\overline{AD} \perp \overline{BC}$,
AB = 13 cm. and BD = 5 cm.
Find : ❶ The length of $\overline{BC}$
❷ The area of Δ ABC



In the opposite figure :
$\overrightarrow{AD}$ bisects $\angle$ BAC ,
AB = AC = 17 cm. ,
and BC = 16 cm.
Prove that : m ($\angle$ ADB) = 90° ,
then find the length of : $\overline{AD}$ and the area of Δ ABC



Inequality

1-Complete

1) The smallest angle of triangle (in measure) is opposite to ……………………..

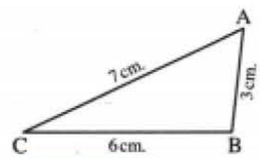
2) The longest side in the right angle triangle is …………………….

3) If triangle ABC m ($\angle$A) =50˚ m ($\angle$B) =30˚

4) If in triangle ABC m ($\angle$A) = m ($\angle$B) +m ($\angle$C) then the longest side in the triangle is ……………..

5) in the triangle ABC if m ($\angle$B) > m ($\angle$C) then ……….< ………….

In the opposite figure :
Arrange the angles of Δ ABC descendingly due to their measures



In the opposite figure :
$\overline{ED} // \overline{BC}$,
AC > AB
Prove that : AE > AD